PARAMETER PLANE STUDIES OF ENGINE-GOVERNOR SYSTEMS WITH TIME DELAY

Francisco José Ferreira Neto

NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

PARAMETER PLANE STUDIES
OF
ENGINE-GOVERNOR SYSTEMS WITH TIME DELAY

by

Francisco José Ferreira Neto

Thesis Advisor: George J. Thaler

December 1972

Parameter Plane Studies
of
Engine-Governor Systems with Time Delay

by

Francisco José Ferreira Neto
Lieutenant, Portuguese Navy
B.S., Portuguese Naval Academy, 1965

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

from the

NAVAL POSTGRADUATE SCHOOL
December 1972

ABSTRACT

The generally difficult problem of a system with time delay is studied by using parameter space methods and computer analysis. The solution is proved to be very easily obtained and the results presented are extendable under certain circumstances to other systems and useful in other fields of analysis.

TABLE OF CONTENTS

I. INTRODUCTION -- 11

II. THEORETICAL BACKGROUND ---------------------------- 13

III. PARAMETER PLANE STUDY OF AN ENGINE-GOVERNOR SYSTEM ---------------------------------- 19

A. GENERAL CONSIDERATIONS ---------------------- 19

B. ENGINE ASSUMPTIONS ------------------------- 20

C. GOVERNOR ASSUMPTIONS ----------------------- 20

D. ENGINE GOVERNOR COMBINATION ---------------- 21

E. USE OF THE PARAMETER SPACE METHOD ---------- 21

IV. TESTING OF THE ENGINE-GOVERNOR SYSTEM ---------- 25

V. PROCESSING OF A PRACTICAL PROBLEM AND ANALYSIS OF RESULTS ------------------------ 27

VI. CONCLUSIONS ------------------------------------ 38

APPENDIX A - DRAWINGS --------------------------------- 39

COMPUTER PROGRAM NUMBER 1 ---------------------------- 110

COMPUTER PROGRAM NUMBER 2 ---------------------------- 119

COMPUTER PROGRAM NUMBER 3 ---------------------------- 120

LIST OF REFERENCES ----------------------------------- 121

INITIAL DISTRIBUTION LIST ---------------------------- 122

FORM DD 1473 --- 123

LIST OF TABLES

I. Parameter settings for systems in which T=0 that guarantee the existence of a pair of complex roots at $-3 \pm j3$ ---------------- 31

II. Parameter settings for systems in which T = 20/450 that guarantee the existence of a pair of complex roots at $-3 \pm j3$ ----------- 31

III. Parameter settings for systems in which T = 40/450 that guarantee the existence of a pair of complex roots at $-3 \pm j3$ ----------- 32

IV. Parameter settings for systems in which T = 60/450 that guarantee the existence of a pair of complex roots at $-3 \pm j3$ ----------- 32

LIST OF DRAWINGS

		Page
1.	Control systems with transport lag in the forward and backward paths ----------------	39
2.	Plot of simultaneous solution of equations (7) and (8) ------------------------------	40
3.	σ_o line in the A = const. plane ---------------	40
4.	σ_o and ω_o lines in A = const. plane -----------	40
5.	σ_o, ω_o line in A, B, C parameter space --------	41
6.	σ_o line in A = const. plane -------------------	41
7.	δ_o plane in A, B, C parameter space -----------	41
8.	Block diagram of the engine-governor system --	42
9.	Transient response of a hypothetical system with zero time delay -------------------	43
10.	Transient response of a hypothetical system with zero time delay -------------------	44
11.	"Best" possible response of a hypothetical system with zero time delay ----------	45
12.	Transient response of a hypothetical system with zero time delay -------------------	46
13.	Projection in the α_3, α_1 parameter plane of the points of the parameter space that guarantee that each of the systems will have a pair of roots at $-3 \pm j3$ -------------------------------------	47
14.	Projection in the α_3, μ_2 parameter plane of the points of the parameter space that guarantee that each of the systems will have a pair of roots at $-3 \pm j3$ --------------	48
15.	Projection in the α_1, μ_2 parameter plane of the points of the parameter space that guarantee that each of the systems will have a pair of roots at $-3 \pm j3$ --------------	49

16. Plot of σ, ω lines (T=0, α_3 = 0.5) ---------------- 50

17. Plot of δ lines (T=0, α_3 = 0.5) ------------------ 51

18. Plot of σ, ω lines (T=0, α_3 = 1.4) --------------- 52

19. Plot of δ lines (T=0, α_3 = 1.4) ------------------ 53

20. Plot of σ, ω lines (T=0, α_3 = 1.5) --------------- 54

21. Plot of δ lines (T=0, α_3 = 1.5) ------------------ 55

22. Plot of σ, ω lines (T=0, α_3 = 1.7) --------------- 56

23. Plot of δ lines (T=0, α_3 = 1.7) ------------------ 57

24. Plot of σ, ω lines (T=20/450, α_3=0.5) ------------ 58

25. Plot of δ lines (T=20/450, α_3=0.5) --------------- 59

26. Plot of σ, ω lines (T=20/450, α_3=1.2) ------------ 60

27. Plot of δ lines (T=20/450, α_3=1.2) --------------- 61

28. Plot of σ, ω lines (T=20/450, α_3=1.3) ------------ 62

29. Plot of δ lines (T=20/450, α_3=1.3)---------------- 63

30. Plot of σ, ω lines (T=20/450, α_3=1.4)------------- 64

31. Plot of δ lines (T=20/450, α_3=1.4)---------------- 65

32. Plot of σ, ω lines (T=20/450, α_3=1.5)------------- 66

33. Plot of δ lines (T=20/450, α_3=1.5)---------------- 67

34. Plot of σ, ω lines (T=20/450, α_3=1.7)------------- 68

35. Plot of δ lines (T=20/450, α_3=1.7)---------------- 69

36. Plot of σ, ω lines (T=40/450, α_3=0.5)------------- 70

37. Plot of δ lines (T=40/450, α_3=0.5)---------------- 71

38. Plot of σ, ω lines (T=40/450, α_3=1.3)------------- 72

39. Plot of δ lines (T=40/450, α_3=1.3)---------------- 73

40. Plot of σ, ω lines (T=40,450, α_3=1.4)------------- 74

41. Plot of δ lines (T=40/450, α_3=1.4)---------------- 75

42. Plot of σ, ω lines (T=40/450, α_3=1.7)------------- 76

43. Plot of δ lines ($T=40/450$, $\alpha_3=1.7$) ---------------- 77

44. Plot of σ, ω lines ($T=40/450$, $\alpha_3=0.5$) ------------- 78

45. Plot of δ lines ($T=60/450$, $\alpha_3=0.5$) ---------------- 79

46. Plot of σ, ω lines ($T=60/450$, $\alpha_3=1.2$) ------------- 80

47. Plot of δ lines ($T=60/450$, $\alpha_3=1.2$) ---------------- 81

48. Plot of σ, ω lines ($T=60/450$, $\alpha_3=1.3$) ------------- 82

49. Plot of δ lines ($T=60/450$, $\alpha_3=1.3$) ---------------- 83

50. Plot of σ, ω lines ($T=60/450$, $\alpha_3=1.4$) ------------- 84

51. Plot of δ lines ($T=60/450$, $\alpha_3=1.4$) ---------------- 85

52. Plot of σ, ω lines ($T=60/450$, $\alpha_3=1.5$) ------------- 86

53. Plot of δ lines ($T=60/450$, $\alpha_3=1.5$) ---------------- 87

54. Plot of σ, ω lines ($T=60/450$, $\alpha_3=1.7$) ------------- 88

55. Plot of δ lines ($T=60/450$, $\alpha_3=1.7$) ---------------- 89

56. Speed transient of Diesel
($\alpha_3=0.5$, $\alpha_1=7.6$, $\mu_2=142.2$) ------------------------- 90

57. Speed transient of Diesel
($\alpha_3=1.4$, $\alpha_1=10.9$, $\mu_2=235.0$) ------------------------- 91

58. Speed transient of Diesel
($\alpha_3=1.42$, $\alpha_1=11.05$, $\mu_2=238.6$) ---------------------- 92

59. Speed transient of Diesel
($\alpha_3=1.5$, $\alpha_1=11.7$, $\mu_2=253.8$) ------------------------ 93

60. Speed transient of Diesel
($\alpha_3=1.7$, $\alpha_1=13.5$, $\mu_2=306.0$) ------------------------ 94

61 Speed transient of 2-cylinder G.E.
($\alpha_3=0.5$, $\alpha_1=9.5$, $\mu_2=150.3$) ------------------------- 95

62. Speed transient of 2-cylinder G.E.
($\alpha_3=1.3$, $\alpha_1=14.6$, $\mu_2=286.0$) ------------------------ 96

63. Speed transient of 2-cylinder G.E.
($\alpha_3=1.34$, $\alpha_1=15.26$, $\mu_2=301.4$) ---------------------- 97

64. Speed transient of 2-cylinder G.E.
($\alpha_3=1.4$, $\alpha_1=16.1$, $\mu_2=320.4$) ------------------------ 98

65. Speed transient of 2-cylinder G.E. (α_3=1.7, α_1=23.8, μ_2=514.8) ---------- 99

66. Speed transient of 4-cylinder G.E. (α_3=0.5, α_1=13.4, μ_2=193.0) ---------- 100

67. Speed transient of 4-cylinder G.E. (α_3=1.2, α_1=24.1, μ_2=438.0) ---------- 101

68. Speed transient of 4-cylinder G.E. (α_3=1.3, α_1=28.5, μ_2=532.0) ---------- 102

69. Speed transient of 4-cylinder G.E. (α_3=1.4, α_1=36.8, μ_2=707.0) ---------- 103

70. Speed transient of 4-cylinder G.E. (α_3=1.7, α_1=421.0, μ_2=9092.0) ---------- 104

71. Speed transient of the 4 systems when the three dominant roots are located at -3 and -3 $\pm$ j3 ---------- 105

72. Speed transient of Diesel (Dominant roots of system at -4 and -4 $\pm$ j4) ---------- 106

73. Speed transient of Diesel (Dominant roots of system at -5 and -5 $\pm$ j5 ---------- 107

74. Roots displacement versus tolerances in the parameters ---------- 108

75. Real root displacement versus tolerances in the parameters ---------- 109

TABLE OF SYMBOLS AND ABBREVIATIONS

N	Rotational speed--RPM
L	Engine load torque--lb-ft
Q	Engine driving torque--lb-ft
Z	Governor actuator position--inches
Δ	Operator, indicating incremental changes
r	Subscript, used to indicate rated conditions
n	per unit speed $\Delta N/N_r$
ℓ	per unit load $\Delta L/L_r$
z	per unit servo stroke $\Delta Z/Z_r$
I	Polar moment of inertia--lb-ft.sec^2
C_6	Constant, with dimensions ft-lbs/inch
T	Dead time--seconds
T_M	Mechanical starting time--seconds
α_1	Inverse of the governor lag time constant
α_2	Governor gain factor
α_3	Inverse of the governor lead time constant
μ_2	$\alpha_2(N_r/Z_r)$
S	Laplacian operator
i	$\sqrt{-1}$
KR	Constant depending on the type of engine
A1	Same as α_1
A3	Same as α_3
MU2	Same as μ_2

ACKNOWLEDGEMENT

I want to express my gratitude to Dr. George Julius Thaler for his attentive assistance in all the phases of this work; his vast knowledge of control theory as well as that of other subjects was a source of constant inspiration and incentive to perfect my skills.

I also extend thanks to Ms. Cynthia S. Carlson for helping me with composition and for typing the rough draft of this thesis.

F.J.F.N.

I. INTRODUCTION

Dead time or transport lag, defined as a fixed time delay in the transmission of information between two segments of a system, is a phenomenon that occurs frequently in every field of engineering. The synthesis of a control system where dead time tends to occur has most discouraging aspects and the first thoughts of the engineer confronted with the problem should be how to go about changing the system design so as to eliminate its sources.

However, many times this is not possible due to physical limitations in the system. The dead time may be due to having the measuring element and the controller actuator necessarily separated by a certain distance, as in the extreme case of the remote control of a space vehicle. In other cases, there is a delay between the time at which the controller actuator is moved and the time at which the plant reacts to the change, as in the case of an engine-governor system.

Dead time deteriorates seriously the stability of a system since it creates a phase lag that constantly increases with ω.

The analytic treatment of the problem is strenuous and obtaining an exact solution is very difficult, if not impossible. The Characteristic Equation (C.E.) of a system with time delay has transcendental nature and a Maclaurin series

of the exponential term shows that the system has an infinite number of roots.

Bode and Root Locus analysis are practical and efficient methods in the study of linear systems, but in the case of a system with transport lag they become too cumbersome. They are also single parameter techniques. However, it is precisely in the case of systems with transport lag that the "Parameter Space Method" (1), (2), appears as a most useful and practical aid. This method has application whenever the system has several adjustable parameters and consists generally in the mapping of the S plane in a N dimensional space of the parameters of the system. With this mapping completed it is possible to choose the N parameter values that allow the system to have a certain set of root locations.

As previously mentioned, a system with transport lag has a C.E. with an infinite number of roots; therefore, if we were attempting to look at stability, the knowledge of the locations of some of the roots would not be the answer. Methods are available for testing the stability of systems with transport lag (2),(3). In this work we will concentrate on finding the parameter values that guarantee a near-optimal behavior of the system recognizing, however, that analysis of stability should be performed after this synthesis phase is finished.

The study of the general problem was followed by application to a practical example.

II. THEORETICAL BACKGROUND

Consider the two block diagrams of Figure 1. The transfer functions of the two systems represented are:

$$T.F._1 = \frac{G(S)e^{-TS}}{1+G(S).H(S).e^{-TS}} \tag{1}$$

$$T.F._2 = \frac{G(S)}{1+G(S).H(S).e^{-TS}} \tag{2}$$

We see that the form of the C.E. of the system does not depend on the location of the dead time block, that is, the C.E. is the same independently of having the dead time block in the forward or backward path. This allows the extension of the following study to a large class of problems where dead time occurs.

The term exp(-TS) gives to this equation a transcendental nature. In fact, the Maclaurin expansion of exp (-TS) is:

$$e^{-TS} = 1 - ST + ST^2 - ST^3 + \ldots\ldots \tag{3}$$

We see then that a system with transport lag has an infinite number of roots which, other than making complete stability analysis very difficult, has the effect of making some graphical methods, as Root Locus and Bode Diagram, very inappropriate.

The stability of systems where dead time appears is always a factor to consider since stability is seriously affected by it. Consider that if we examine e^{-TS} through

sinusoidal analysis and let $s = j\omega$, the transport lag term becomes

$$F(j\omega) = e^{-j\omega T}$$

$$= |1| \; e^{-j\omega T} \quad . \qquad (4)$$

We see then that the transport term has a magnitude of 1 and a lag phase angle of ωT. The phase lag added by the transport lag term increases as ω increases. The system may, therefore, be very difficult to stabilize.

However, we assumed initially that the nature of the system is such that all the roots lie in the left half of the S plane. The system is then intrinsically stable and the problem is to find the values of the adjustable parameters that make the system behave in an optimal manner in relation to an established criterion. We concentrated on finding the parameter values that guarantee the existence of a set of dominant roots for the system, that is, a set of roots close to the origin. Stability analysis, using for instance, the methods in references (2) and (3), should be used after the parameters for the system were selected. To determine the parameter values, the parameter space method can be used with great advantage since, as will be shown, it allows us to relate precisely parameter values with root locations. We considered the case of systems with three adjustable parameters.

The general form of the C.E. of a system with transport lag is:

$$F(S) = \sum_{i=0}^{N} K_i S^i + \sum_{j=0}^{N} K_j S^j e^{-TS} = 0 \qquad (5)$$

For $\quad S = -\sigma \pm j\omega$

$$e^{-TS} = e^{T\sigma \mp jT\omega} = e^{T\sigma} (\cos T\omega \mp j \sin T\omega) \qquad (6)$$

This transformation allows the separation of the C.E. into two equations, one containing the real and the other the imaginary part. That is:

$$F(\sigma,\omega) = F_e(\sigma,\omega) + jF_o(\sigma,\omega) = 0$$

implies that:

$$F_e(\sigma,\omega) = 0 \qquad (7)$$

$$F_o(\sigma,\omega) = 0 \qquad (8)$$

If we consider that the system has three adjustable parameters A, B and C, then they appear in $F_e(\sigma,\omega)$, and/or $F_o(\sigma,\omega)$. Fixing one of the parameters (for instance, parameter A), equations (7) and (8), may be solved for the two parameters B and C. Recapitulating, we have that equations (7) and (8) are functions of the variable parameters A, B and C of σ and ω from the assumed roots $-\sigma \pm j\omega$. Fixing all these variables but two of the parameters, for instance, B and C, simultaneous solution of the two equations allows the determination of the two values B_o and C_o, that together with the fixed value of A_o guarantees that the system has a pair of conjugate roots at $-\sigma_o \pm j\omega_o$.

Repeating this process for all different values of ω in a certain range permits the determination of different set-pairs, B_oC_o, as in Figure 2. Of course, there is an infinite number of these solutions. The union of these solutions B_oC_o is a line in the parameter plane of axis B and C as is shown in the Figure 3 (σ_o line). Working similarly, but with the variation of σ instead of ω, it is possible to obtain similar lines in the parameter plane B-C for which ω is fixed (ω lines).

The intersection of a certain line of fixed ω_o(ω_o line) and another line of fixed σ_o (σ_o line) gives a point in the parameter plane from which it is possible to read the pair of parameters $B_o^* C_o^*$ that guarantee the existence of a pair of roots for the system of the form $-\sigma_o \pm j\omega_o$. See Figure 4.

Repeating the above process for different values of A allows the determination of a line (or a group of points) in the three-dimensional space with ordinates A, B and C for which every point corresponds to assigning the system the pair of roots $-\sigma_o \pm j\omega_o$. See Figure 5.

If we assume that the system has a third dominant root $-\delta_o$- of a real nature, we can, fixing the value of A, determine a line of fixed δ(δ_o line) in the parameter plane B-C as in Figure 6. This is so because if we substitute the solution δ_o in the characteristic equation and fix its value together with the value of A, then this equation becomes the equation of a line in the plane B-C.

Assuming different values for A, but keeping δ constant, the repetition of this process for each value of A results in a group of lines in which δ is constant, and these lines, if this repetition were performed for an infinite number of values of A, form a surface (surface δ_o) in the parameter space A, B and C for which the system has a real root δ_o, as in Figure 7.

In the three-dimensional parameter space, the pierce point(s) of the line $-\sigma_o \pm j\omega_o$ with the plane $-\delta_o$ gives one or more points that correspond to the set(s) of parameters A_o, B_o and C_o, for which the system has a set of three roots $-\sigma_o \pm j\omega_o$ and δ_o that we assumed would be the dominant ones.

Notice that, since we are mostly interested in finding roots that lie close to the origin, the range of variation of σ, ω, and δ during the process of mapping the S-plane in the parameter space does not need to be very large.

Of course, such an extensive manipulation of equations and plot of curves requires the use of a computer.

We will proceed now to apply these theoretical considerations to the study of a practical problem, specifically, to the determination of the set of near-optimal parameters to be used in the operation of an engine-governor system. This problem was suggested by the reading of reference (4) and subsequent exchange of correspondence with one of its authors.

In this paper, it is suggested that the determination of the parameter settings must be made by assuming that the dominant roots of the system occupy a certain well-defined

position, specifically that the real and imaginary parts of the complex roots must have equal magnitude equal also to the magnitude of the third (real) root.

The parameter space method was used to find new combinations of parameter values that would permit us to determine if the root locations, as assumed in the aforementioned paper, would correspond to an optimal solution, and to search for new solutions. In addition, the results permit study of related problems such as the problem of sensitivity.

III. PARAMETER PLANE STUDY OF AN ENGINE-GOVERNOR SYSTEM

A. GENERAL CONSIDERATIONS

Most gas engines exhibit a certain degree of dead time, that is, an interval of time between a change in fuel flow, and the corresponding change in driving torque. This time lag is made of a number of components of which the most important are: the delay in "sensing" the speed change that follows the load change; the delay within the governing mechanism during which the reaction of the sensing element is being translated into governor action; the time required for the governor action to correct the fuel injection pump setting; the delay that must take place before the corrected fuel change can be introduced into an engine cylinder; the time required to convert the corrected fuel change into engine torque.

Dead time is found to be detrimental to stable operation and tends to increase speed transient errors. It also determines the maximum rate at which the engine-governor combination can be operated. The purpose of this study is to find the best isochronous governor parameters so that optimum transients, for the operation of gas engines of several types be obtained.

The study of ideal governor settings in the presence of dead time will be made in the parameter space. This method leads to a computer process for analyzing the variable system parameters. The process is restricted to a two parameter space.

B. ENGINE ASSUMPTIONS

The driving torque incremental change ΔQ is related to the small variations of the governor actuator ΔZ by the equation:

$$\Delta Q = C_6 \, e^{-TS} \, \Delta Z \tag{9}$$

where e^{-TS} is the dead time operator.

The minimum amount of dead time T that can be expected is given by the following formulae:

$$T = 20/N \quad \text{for a Diesel engine} \tag{10}$$

$$T = 40/N \quad \text{for a two-cycle gas engine (G.E.)} \tag{11}$$

$$T = 60/N \quad \text{for a four-cycle gas engine (G.E.)} \tag{12}$$

in which N is the rotational speed in RPM.

The assumptions made relative to the engine rotating parts are that the accelerating torque is equal to the difference between the load torque and the driving torque as expressed by the following equation:

$$\frac{\pi I}{30} S \, \Delta N = \Delta Q - \Delta L \tag{13}$$

Combination of (9) and (13) gives:

$$\frac{\pi I}{30} S \, \Delta N = C_6 \, e^{-TS} \, \Delta Z - \Delta L \tag{14}$$

Change to a system of per unit variables gives the engine equation used in the analysis:

$$T_M \, Sn = e^{-TS} Z - \ell \tag{15}$$

C. GOVERNOR ASSUMPTIONS

It is assumed that the isochronous pressure compensated type of governor is used. The differential equation

describing its dynamics is:

$$(S^2 + \alpha_1 S)Z = -\mu_2(S + \alpha_3)n \tag{16}$$

where $\mu_2 = (N_r/Z_r)\alpha_2$.

D. ENGINE GOVERNOR COMBINATION

Combination of equations (15) and (16) gives:

$$(S^3 + \alpha_1 S^2 + \frac{\mu_2}{T_M} e^{-TS} S + \frac{\mu_2\alpha_3}{T_M} e^{-TS})n = -\frac{1}{T_M} S(S+\alpha_1)\ell \tag{17}$$

E. USE OF THE PARAMETER SPACE METHOD

We start with the engine-governor equation (Equation 17):

$$(S^3 + \alpha_1 S^2 + \frac{\mu_2}{T_M} e^{-TS} S + \frac{\mu_2\alpha_3}{T_M} e^{-TS})n = -\frac{1}{T_M} S(S+\alpha_1)\ell$$

From equation (17), the C.E. can be extracted:

$$S^3 + \alpha_1 S^2 + \frac{\mu_2}{T_M} e^{-TS} S + \frac{\mu_2\alpha_3}{T_M} e^{-TS} = 0 \tag{18}$$

Since we have three adjustable parameters in the characteristic equation, we can control the position of three of its roots. We assumed then that the system had three dominant roots similarly to what would happen if the delay in the same equation were equal to zero. For these three dominant roots we had two choices with respect to their intended location, that is, we could choose to have three real roots, or two complex conjugate roots and a real one. Because we were interested in having a small rise time for the transients of the system, we chose the second case, that is, we assumed that the system should exhibit a pair of complex

conjugate roots and a single real root close to the origin in the S plane.

In accordance with this choice we made:

$$S = -\sigma + j\omega$$

We have then that:

$$S^2 = (\sigma^2 - \omega^2) - j(2\sigma\omega) \tag{19}$$

$$S^3 = (3\sigma\omega^2 - \sigma^3) + j(3\sigma^2\omega - \omega^3) \tag{20}$$

The exponential term e^{-TS} becomes:

$$\begin{aligned} e^{-TS} &= e^{-T(-\sigma+j\omega)} \\ &= e^{T\sigma}\, e^{-j\omega T} \\ &= e^{T\sigma}\,(\cos\omega T - j\sin\omega T) \end{aligned} \tag{21}$$

Substituting (19), (20) and (21) in (18) gives:

$$\begin{aligned} &(3\sigma\omega^2 - \sigma^3) + j(3\sigma^2\omega - \omega^3) + \alpha_1(\sigma^2 - \omega^2 - j2\sigma\omega) \\ &+ \frac{\mu_2}{T_M}\, e^{T\sigma}(\cos\omega T - j\sin\omega T)(-\sigma + j\omega) \\ &+ \frac{\mu_2\alpha_3}{T_M}\, e^{T\sigma}(\cos\omega T - j\sin\omega T) = 0 \end{aligned} \tag{22}$$

Both the real and the imaginary parts of this equation must be equal to zero:

$$3\sigma\omega^2 - \sigma^3 + \alpha_1(\sigma^2 - \omega^2) + \frac{\mu_2}{T_M} e^{T\sigma}(\omega\sin\omega T - \sigma\cos\omega T + \alpha_3\cos\omega T) = 0 \tag{23}$$

$$3\sigma^2\omega - \omega^3 - 2\alpha_1\sigma\omega + \frac{\mu_2}{T_M} e^{T\sigma}(\sigma\sin\omega T + \omega\cos\omega T - \alpha_3\sin\omega T) = 0 \tag{24}$$

Equations (23) and (24) form a system that can give solutions for any two parameters if the others are fixed. If we take α_1 and μ_2 for parameters of the plane, and fix the value of α_3, we can get a complete representation of the loci of

the points $\left(\alpha_1(\omega), \mu_2(\omega), \sigma = \text{constant or } \alpha_1(\sigma), \mu_2(\sigma), \omega = \text{constant}\right)$ for which the C.E. has a solution of the form $-\sigma + j\omega$.

The curves determined as above-curves σ and ω are loci of all parameter pairs (α_1 and μ_2) for which the solutions $-\sigma + j\omega$ and $-\sigma - j\omega$ exist.

If we denote by δ the dominant real root of the C.E., we can by the same methods get a plot of δ lines in the α_1, μ_2 parameter plane.

We can repeat the process of finding σ, ω and δ curves for as many values of α_3 as we want.

For a certain α_3 = constant, the intersection of selected σ_o and ω_o curves is the loci of values of the parameters α_1 and μ_2 for which the selected solution $-\sigma_o \pm j\omega_o$ exists.

In the three dimensional orthogonal space (α_1, μ_2 and α_3) the loci of pairs α_1 and μ_2 is a line along which it is guaranteed that any readings of α_1, μ_2 and α_3 guarantee the existence of the above form of solution. In a similar way, the set of lines of constant δ_o, one for each value of α_3 forms a plane in the three dimensional space.

The single or multiple intersections of the aforementioned line with the plane of constant $\delta = \delta_o$ gives the set(s) of parameters α_1, μ_2 and α_3 for which the system has a real root δ_o and a pair of conjugate roots $-\sigma_o \pm j\omega_o$. However, since we are primarily interested in finding the near-optimal set of parameters for the governor, that is, the set of parameters that offers a "better" transient, we will not be

looking for a set of the three parameter space parameters as indicated above. Instead, we will fix the location of the pair of complex roots, which, as explained above, will allow the determination of a line in the three dimensional parameter space. We will then proceed to test the system with parameters corresponding to points of this line. Of course, the parameters corresponding to each of these points determine a different location for the dominant real root.

A computer program was written to plot the σ, ω and δ curves on a parameter plane with axis α_1 and μ_2 for a number of values of the parameter α_3 (computer program #1). This program was designed with the purpose of making it adaptable to studies of any type of engine-governor combination that may be described by equation (17).

We performed the computer analysis first for a hypothetical system in which the transport lag was zero and later for each of the three practical cases considered--Diesel engine, two-cycle gas engine, and four-cycle gas engine--each of them with the same C.E., but with different dead times as in equations (10), (11) and (12). The problem of obtaining parameter values that guarantee certain root locations is thus solved. We still needed, however, any kind of way of testing the systems with each of the sets of parameters obtained through the parameter space computer program. We, naturally, made use of a second computer program (computer program #2) which was designed under the following heading.

IV. TESTING OF THE ENGINE-GOVERNOR SYSTEM

We want a program to test each of the systems with different groups of parameter settings. From equation (17) we determine the transfer function (T.F.):

$$\frac{n}{\ell} = \frac{-\frac{1}{T_M} S(S+\alpha_1)}{S^3 + \alpha_1 S^2 + \frac{\mu_2}{T_M} e^{-TS} S + \frac{\mu_2 \alpha_3}{T_M} e^{-TS}} \tag{25}$$

The denominator of the above T.F. is rearranged as follows:

$$\begin{aligned} S^3 + \alpha_1 S^2 + \frac{\mu_2}{T_M} e^{-TS} S + \frac{\mu_2 \alpha_3}{T_M} e^{-TS} \\ &= S^2 (S+\alpha_1) + \frac{\mu_2}{T_M} e^{-TS} (S+\alpha_3) \\ &= S^2 (S+\alpha_1)\left(1 + \frac{\mu_2}{T_M} \frac{e^{-TS}(S+\alpha_3)}{S^2 (S+\alpha_1)}\right) \end{aligned}$$

Then, the T.F. becomes:

$$\begin{aligned} \frac{n}{\ell} &= \frac{-\frac{1}{T_M} S(S+\alpha_1)}{S^2 (S+\alpha_1)\left(1 + \frac{\mu_2}{T_M} \frac{e^{-TS}(S+\alpha_3)}{S^2 (S+\alpha_1)}\right)} \\ &= \frac{-\frac{1}{T_M} S}{1 + \frac{\mu_2}{T_M} \frac{e^{-TS}(S+\alpha_3)}{S^2 (S+\alpha_1)}} = \frac{G}{1+GH} \end{aligned} \tag{26}$$

This implies that: $$G = -\frac{1}{T_M S} \tag{27}$$

$$H = -\frac{\mu_2 e^{-TS}(S+\alpha_3)}{S(S+\alpha_1)} \tag{28}$$

Using block diagram description, we can partition G and H in the following form:

$$G = \longrightarrow \boxed{-\frac{1}{T_M}} \longrightarrow \boxed{\frac{1}{S}} \longrightarrow$$

$$H = \longrightarrow \boxed{e^{-TS}} \longrightarrow \boxed{\frac{1}{S}} \longrightarrow \boxed{\frac{1/\alpha_3 S+1}{1/\alpha_1 S+1}} \longrightarrow \boxed{-\frac{\mu_2\alpha_3}{\alpha_1}} \longrightarrow$$

And the complete block diagram of the feedback system becomes as represented in Figure 8. This configuration allows the use of C.S.M.P.

V. PROCESSING OF A PRACTICAL PROBLEM AND ANALYSIS OF RESULTS

The design of governors requires the choice of a "standard" type of response to load changes. Since a change in load unavoidably causes a transient change in speed, the usual application has the following requirements:

a) Recovery should be fast and with negligible difference in steady state speed;

b) The maximum deviation from steady state speed should be less than some specified percent;

c) The transient recovery should not overshoot the steady state value.

Trade-offs between these requirements may be necessary in some applications.

For a system with no time delay, but with third order characteristics, the best solution for items a) and c) is a set of two complex and one real root, such that the real root is numerically equal to the real parts of the complex roots, and such that the imaginary and real parts of the complex roots are equal. For typical applications, a numerical choice which gives a satisfactory solution is a real root at -3 and complex roots at $-3 \pm j3$. (A study of results based on other choices is made later.) Justification of this is given by Figures 9, 10, 11 and 12, which show the transient responses for a system with no time delay, with complex roots at $-3 \pm j3$ and with several locations of the

real root. Notice that Figure 11 is the one that corresponds to having the real root δ equal to -3 and that Figures 9 and 10 don't satisfy item a) while Figure 12 shows an overshoot which goes against the requirement posed in item c).

We wanted to find the near-optimal parameter settings for the governor described in reference 4 for each of the types considered in the same paper. We assume in all cases that the steady state speed of the engine is 450 RPM.

We used some practical data as it appears in the same reference:

$$T_M = 5.88$$

$$T = KR/N = 20/450 \text{ for a Diesel engine}$$

$$= 40/450 \text{ for a 2 cycle G.E.}$$

$$= 60/450 \text{ for a 4 cycle G.E.}$$

We needed also to know what was the behavior of a system governed by equation (17), but with zero delay. This could be helpful in establishing the effect of the time delay and was used in determining parameter values that, entered as data in computer program #2, allowed the plotting of Figures 9, 10, 11 and 12.

In each case the parameter plane program was run for values of the parameter α_3 equal to 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.55, 1.6 and 1.7.

We decided in all cases that this system should have a pair of roots at $-3 \pm j3$ and so we concentrated on finding the location of the point of the parameter planes where the curves $\omega = 3.0$ and $\sigma = 3.0$ intersected. Had the choice of

root locations been different, we could still have used the same parameter plane plots since they contain curves for values of σ and ω equal to 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0, 5.5 and 6.0. The reason why we did not run the program for values of α_3 higher than 1.7 was that we found that in each case for values of α_3 higher than a well-determined value, the $\sigma = 3.0$ and $\omega = 3.0$ curves on which we concentrated, would not cross in the parameter plane. This is justified by the following:

For example, in the case of the four cycle gas engine with $\sigma = \omega = 3.0$, we have from equation (23) that:

$$54 + \mu_2 \times 0.17 \times e^{0.4}(3 \sin 0.4 - 3 \cos 0.4 + \alpha_3 \cos 0.4) = 0$$

$$.254\ \mu_2\ (0.92\ \alpha_3 - 1.59) = -54$$

$$\mu_2 = \frac{-212.6}{.92\alpha_3 - 1.59}$$

Assuming that the physical nature of the parameters requires that they have positive values has the effect that μ_2 must be positive, which implies that

$$\alpha_3 < \frac{1.59}{.92} \simeq 1.73$$

The analysis of the parameter plane plots proceeded as follows: for each α_3 = const., we determined the intersection of the $\sigma = 3.0$ and $\omega = 3.0$ curves and read the corresponding coordinate values of α_1 and μ_2.

Plotting the point correspondent to these parameter values in page 2 of the parameter plane plot, (page of curves of constant δ) allowed the determination of the location of

the real root for the considered value of α_3. This value of δ was further checked by using computer program #3.

Tables 1, 2, 3 and 4 list in each row the parameter values obtained by this process, and that correspondingly must be used in testing the dynamics of each of the systems so that they present a pair of complex roots at $-3 \pm j3$, and the value of the real root determined as described.

These tables were used to plot Figures 13, 14 and 15 that represent the projections in each of the orthogonal planes that form the parameter space of the points of this space that guarantee that each of the systems will have a pair of roots at $-3 \pm j3$. Notice in these plots that the points correspondent to a certain system are disposed "in a line". Notice also that due to this form of disposition, if we were to work with a system described by equation (17), but with a delay different from those considered here, the determination of parameter values that would give the same root locations would be possible by simply using interpolation.

Figures 16 through 55 are some of the parameter plane plots obtained by using the parameter plane program in the IBM 360/65 digital computer. These are only a part of the large number of parameter plane plots that we had to obtain to construct Tables 1, 2, 3 and 4. We chose those that either give parameter values close to those found to the "best" or the ones that give real "bad" parameter sets.

α_3	α_1	μ_2	δ
0.5	6.6	126.9	-0.60
0.6	6.7	132.3	-0.75
0.7	6.9	138.6	-0.91
0.8	7.1	145.8	-1.09
0.9	7.2	150.0	-1.29
1.0	7.4	158.0	-1.49
1.1	7.5	167.0	-1.68
1.2	7.9	177.0	-1.95
1.3	8.3	188.0	-2.28
1.4	8.5	198.2	-2.54
1.49	8.97	209.5	-3.00
1.5	9.0	210.6	-3.03
1.55	9.2	216.9	-3.26
1.6	9.3	225.0	-3.33
1.7	9.8	242.0	-3.82

Table 1 - T = 0

α_3	α_1	μ_2	δ
0.5	7.6	142.2	-0.60
0.6	7.8	144.5	-0.77
0.7	8.2	148.5	-0.95
0.8	8.5	154.8	-1.15
0.9	8.7	165.0	-1.29
1.0	9.0	175.0	-1.58
1.1	9.5	187.0	-1.88
1.2	10.0	201.6	-2.18
1.3	10.3	217.0	-2.43
1.4	10.9	235.0	-2.82
1.42	11.05	238.6	-3.00
1.5	11.7	253.8	-3.45
1.55	12.0	264.0	-3.68
1.6	12.4	279.0	-3.85
1.7	13.5	306.0	-4.88

Table 2 - T = 20/450

α_3	α_1	μ_2	δ
0.5	6.6	126.9	-0.60
0.6	6.7	132.3	-0.75
0.7	6.9	138.6	-0.91
0.8	7.1	145.8	-1.09
0.9	7.2	150.0	-1.29
1.0	7.4	158.0	-1.49
1.1	7.5	167.0	-1.68
1.2	7.9	177.0	-1.95
1.3	8.3	188.0	-2.28
1.4	8.5	198.2	-2.54
1.49	8.97	209.5	-3.00
1.5	9.0	210.6	-3.03
1.55	9.2	216.9	-3.26
1.6	9.3	225.0	-3.33
1.7	9.8	242.0	-3.82

Table 1 - T = 0

α_3	α_1	μ_2	δ
0.5	7.6	142.2	-0.60
0.6	7.8	144.5	-0.77
0.7	8.2	148.5	-0.95
0.8	8.5	154.8	-1.15
0.9	8.7	165.0	-1.29
1.0	9.0	175.0	-1.58
1.1	9.5	187.0	-1.88
1.2	10.0	201.6	-2.18
1.3	10.3	217.0	-2.43
1.4	10.9	235.0	-2.82
1.42	11.05	238.6	-3.00
1.5	11.7	253.8	-3.45
1.55	12.0	264.0	-3.68
1.6	12.4	279.0	-3.85
1.7	13.5	306.0	-4.88

Table 2 - T = 20/450

α_3	α_1	μ_2	δ
0.5	9.5	150.3	-0.63
0.6	9.7	157.5	-0.78
0.7	10.2	167.0	-1.02
0.8	10.7	182.0	-1.19
0.9	11.3	196.0	-1.44
1.0	11.9	213.3	-1.70
1.1	12.8	234.0	-2.04
1.2	13.8	256.5	-2.50
1.3	14.6	286.0	-2.72
1.34	15.26	301.4	-3.00
1.4	16.1	320.4	-3.36
1.5	17.8	367.2	-3.87
1.55	18.9	390.0	-4.25
1.6	20.3	430.2	-4.72
1.7	23.8	514.8	-6.80

Table 3 - T = 40/450

α_3	α_1	μ_2	δ
0.5	13.4	193.0	-0.67
0.6	12.7	207.0	-0.90
0.7	13.4	225.0	-1.02
0.8	15.4	251.0	-1.25
0.9	16.8	280.0	-1.58
1.0	18.4	317.0	-1.91
1.1	21.0	368.0	-2.15
1.2	24.1	438.0	-2.54
1.3	28.5	532.0	-3.00
1.4	36.8	707.0	-3.69
1.5	48.4	971.0	-4.50
1.55	65.3	1300.0	-4.95
1.6	89.5	1859.0	-5.50
1.7	421.0	9092.0	-7.20

Table 4 - T = 60/450

Each of the 3 systems was simulated including their respective time delays and the parameter values as in each row of tables 1-4. Figures 56 through 70 are plottings of the transients of each of the systems when subjected to 100% of load change. We obtained the transient responses for every set of parameters in those tables, but we are including only as many as needed to illustrate the following analysis.

Figure 71 is a plot of the transients of each of the systems for the case in which the dominant real root has a value of -3.0. We can see that all these transient responses are very similar. Thus the three roots considered in the cases of the systems in which the delay is non-zero, are really dominant over all the others, and the determination of the near-optimal parameters for a system with transport lag consists of finding the parameter values that determine the location of the dominant roots to be the ideal location of the roots for a system with the same equation, but with zero time delay. (This criteria is restricted to a certain portion of the S plane as will be seen later.)

Examination of Figure 71 shows that for each of the systems, the settling time is very approximately the same; the differences in time delay determine only the amplitude of the transient, this being proportional to the amplitude of the delay, which makes sense since the greater the delay, the more time the governor takes to interpret the change and set the needed corrections.

A comparative analysis of Figures 56 through 70 shows that the transients of each system with parameter settings that correspond to having the real root closer to the origin, have larger settling times, while in the opposite case, the recoveries are faster, but an overshoot appears.

The study of the method of determination of parameter sets is completed. We tried to make use of the large amount of data available by proceeding with the analysis of some other subjects. The first of these relates to the choice of location of the pair of complex roots.

For a <u>linear</u> system, we know that the same root pattern, that is, having the real root numerically equal to the real part of the complex roots and this real part equal in magnitude to the imaginary part, should give the same type of transient, except of course for time scaling. In other words, the three groups of dominant roots,

$$\begin{pmatrix} -3 \\ -3 \pm j3 \end{pmatrix}, \quad \begin{pmatrix} -4 \\ -4 \pm j4 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} -10 \\ -10 \pm j10 \end{pmatrix}$$

should all give identical transients for a <u>linear</u> system, except for the time scale. However, <u>in the case of a system with time delay, this should not apply, and there should be some numerical limit on the pattern, depending on the value of T</u>. In other words, if we tried to make this root pattern fall far to the left of the S plane, either the parameters would become negative-therefore physically unrealizable-or some other roots would become dominant, or would even fall in the right half of the S plane. The answer to this should

be looked at, because, given that this pattern were realizable more to the left of the S plane, certainly the transient would become faster, and the magnitude of the maximum deviation could also possibly be smaller.

We started by finding the sets of parameters that would give to the Diesel governor system root sets respectively at (-4 and -4 ± j4), and (-5 and -5 ± j5). The determination of parameter sets went as before, that is, in each case, we looked in page one of each of the plots of const. α_3 for the intersections of the $\sigma = 4$ with $\omega = 4$ and $\sigma = 5$ with $\omega = 5$. For each of these intersections, we determined in page two of the plot the corresponding value of the real root δ, and were finally able to determine the values of the parameters α_1, α_3 and μ_2, that guarantee the desired root locations. The parameter sets determined are displayed in the following table:

DOMINANT ROOTS	α_1	α_3	μ_2
-4, -4+j4	16.16	1.84	444.5
-5, -5+j5	20.91	2.25	643.0

We used computer program #2 to determine the responses of the system while working with these sets of parameters, and we obtained the transients displayed in Figures 72 and 73. Looking at Figure 72, it can be seen that, with the parameters set such that the system has dominant roots at -4 and -4 ± j4, the speed transient is faster and of smaller

amplitude than any one of those previously found, which guarantees that the correspondent choice of parameters is closer to the "optimal". However, the response, displayed in Figure 73 of the system with dominant roots at -5 and $-5 \pm j5$, is not good under item c) of the established criteria. Here the recovery is very fast and the amplitude of the transient is even smaller but an overshoot shows. This proves that our hypothesis about the existence of a limit in the process of bringing the root pattern more to the left of the S plane, was reasonable.

Another topic of interest was the one related to tolerances in the parameter values and/or sensitivity of the systems to parameter change. The possibility of and ease in performing such studies is one of the great advantages of the parameter space method, since the parameter plane plots display sets of curves that allow to relate changes in parameter values with changes in root location. Figures 74 and 75 show a parameter plane plot where we marked the point that confers to the engine-governor system a set of roots at -A and $-A \pm jA$. The -.- lines correspond to variations of a certain percent in the determined values of the parameters α_1 and μ_2. The possible changes in root locations, due to changes in parameter values, is quite obvious. For example, the considered variation in the value of α_1, while μ_2 is kept constant, can cause variation of ω between D and E and variations in the values of σ between B and C. Obviously, similar considerations apply to page 2 of each

plot and allow the study of changes in the value of δ. The analysis of the effects of changes in α_3 is a little more difficult, since it requires the use of plots correspondent to adjacent values of α_3. Here we should plot the point correspondent to the nominal values of α_1 and μ_2 in the adjacent plots. The point plotted in the adjacent plots shows immediately the changes in σ, ω and δ due to the variation of α_3.

Of course, if it were desirable to do so, the whole process could be reversed; that is, we could easily find out the new values of the parameters, if it were required to change the location of the roots in relation to the values set initially.

Finally, notice that the parameter plane plots are a great aid in studies of sensitivity of the systems since the slope of the lines plotted gives direct measure of that characteristic.

VI. CONCLUSIONS

The association of the parameter plane method, with appropriate computer analysis, converted a rather difficult problem into an almost elementary one. In effect, after passing the difficulties in writing and debugging the parameter plane program, the analysis of the information available is so easy, that an ill-advised observer may not notice all the trouble involved in the solution of the same problem, if it were attempted by other methods.

The problem of finding the set of near optimal parameters with time delay consists of finding the parameter values that guarantee that the dominant roots of the C.E. of the system will be located according to a special pattern. The quality of the response improves with the displacement of the dominant roots to the left of the S plane but, this process has a limit depending on the value of the delay. The determination of this limit at which the quality of the response should be near-optimal has to be made through a trial and error process.

The method described in this study is very easy to follow, and the information available allows for investigation in many areas of interest, such as sensitivity of the systems to variations in parameter values, analysis of permitted tolerances in these values, optimization of performance under a defined cost function, and others.

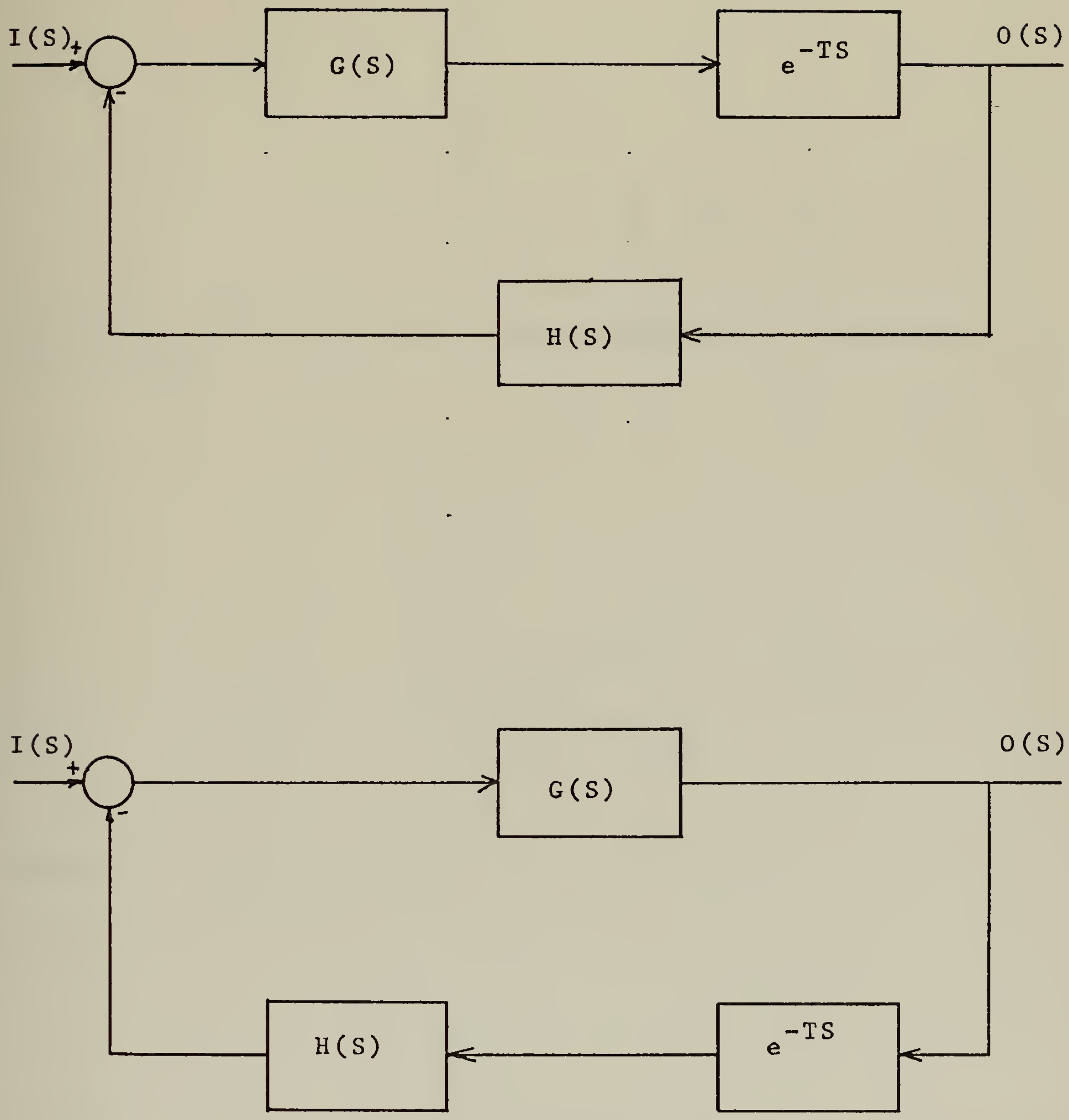


Figure 1. Control systems with transport lag in the forward and backward paths.

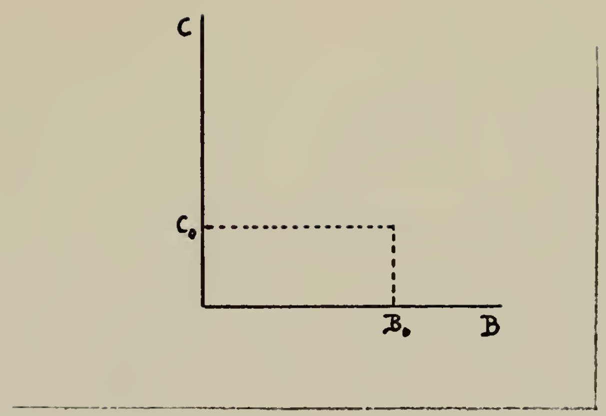


Figure 2. Plot of simultaneous solution of equations (7) and (8).

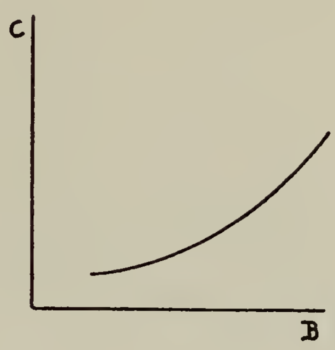


Figure 3. σ_o line in the A = const. plane.

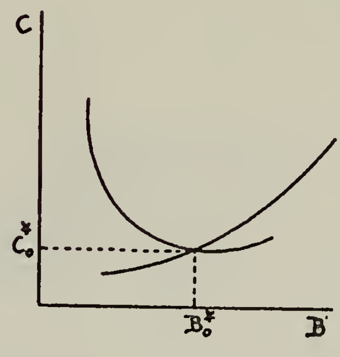


Figure 4. σ_o and ω_o lines in A = const. plane.

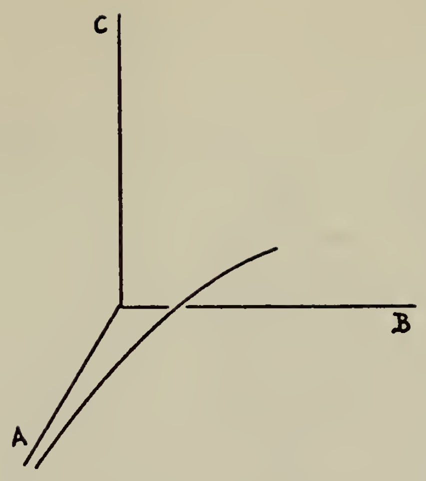


Figure 5. σ_o, ω_o line in A, B, C parameter space.

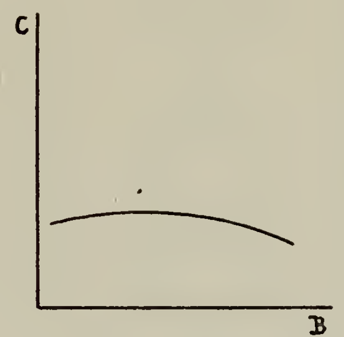


Figure 6. σ_o line in A = const. plane.

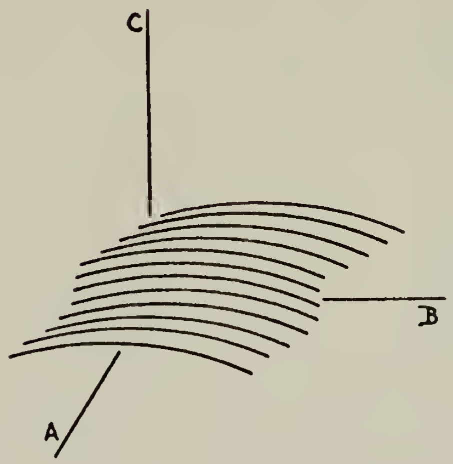


Figure 7. δ_o plane in A, B, C parameter space.

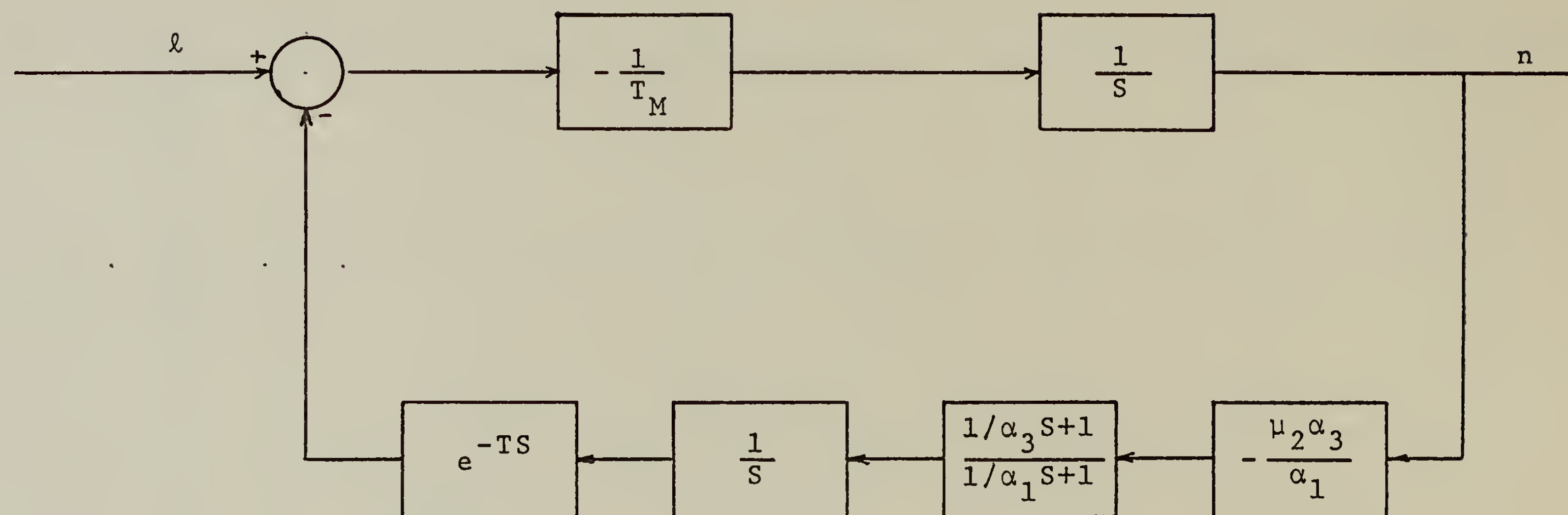


Figure 8. Block diagram of the engine-governor system.

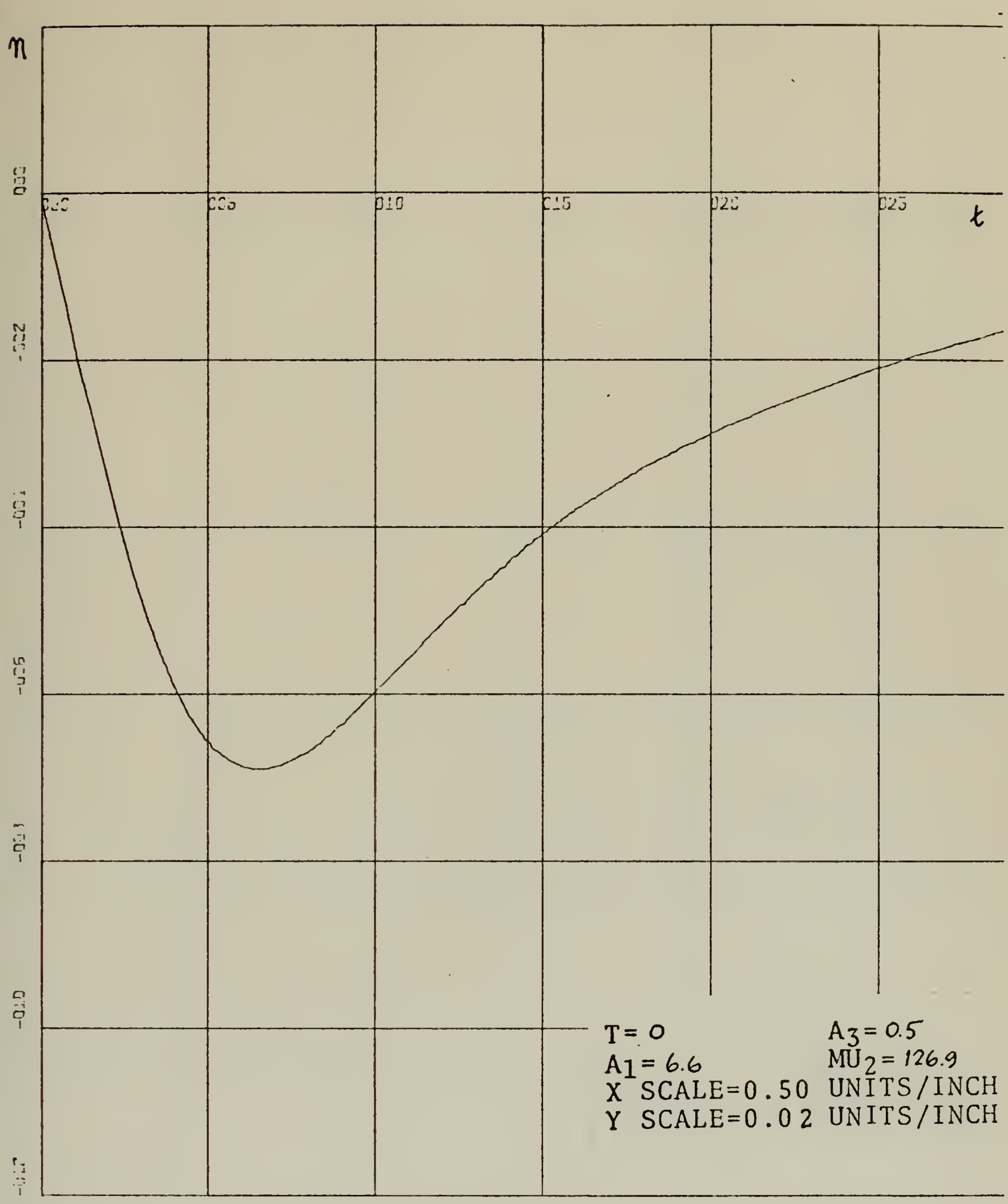


Figure 9. Transient response of a hypothetical system with zero time delay.

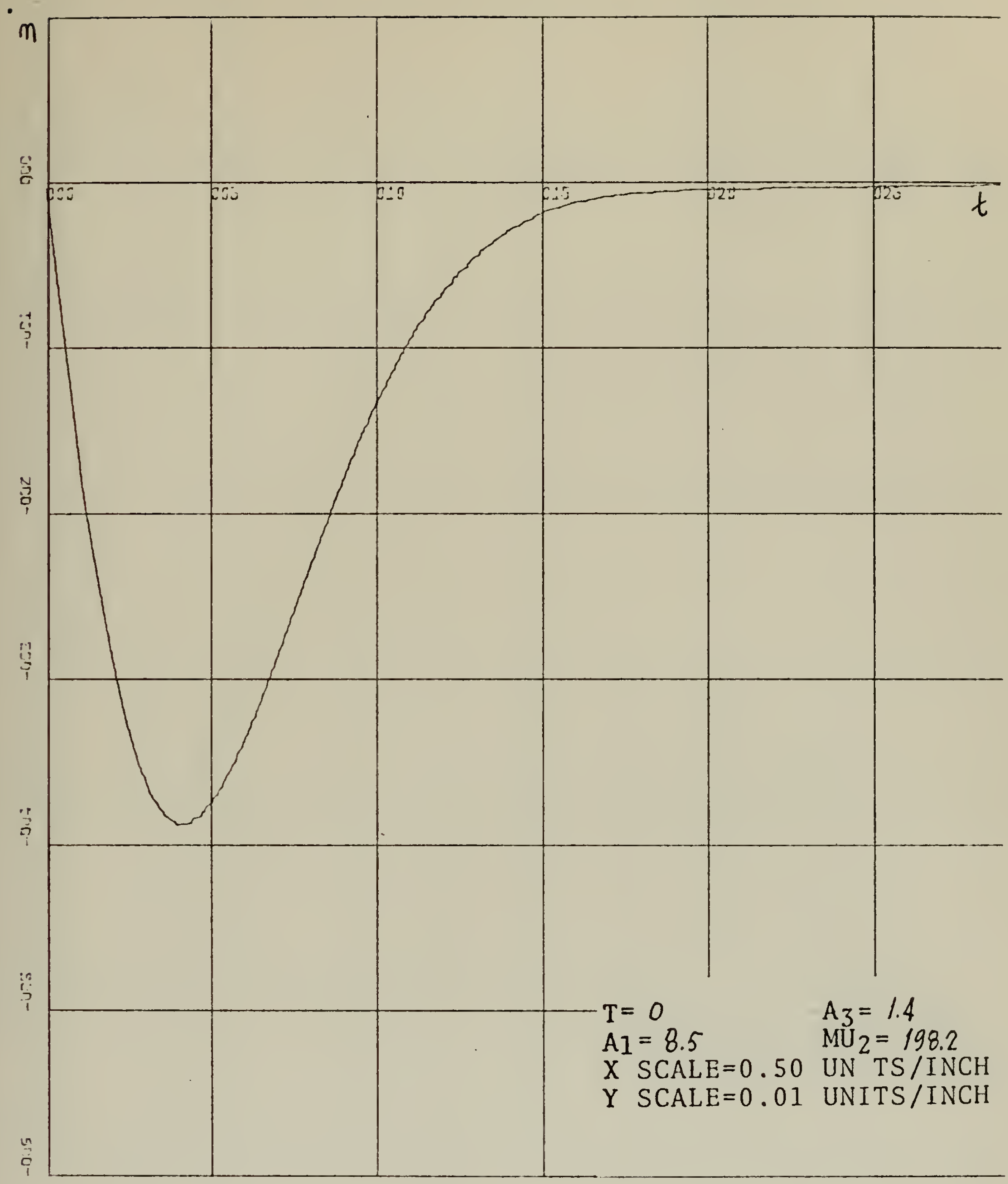


Figure 10. Transient response of a hypothetical system with zero time delay.

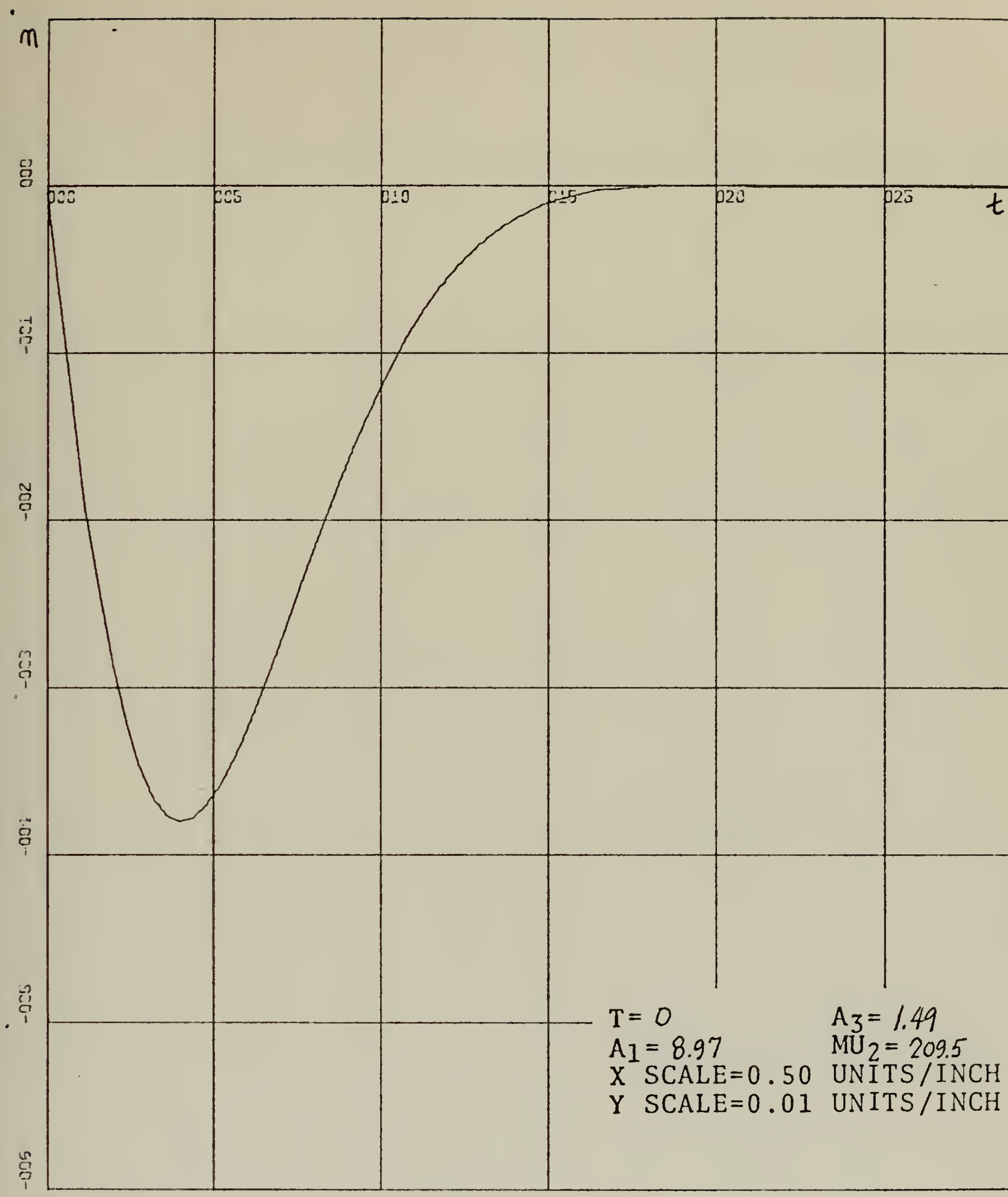


Figure 11. "Best" possible response of a hypothetical system with zero time delay.

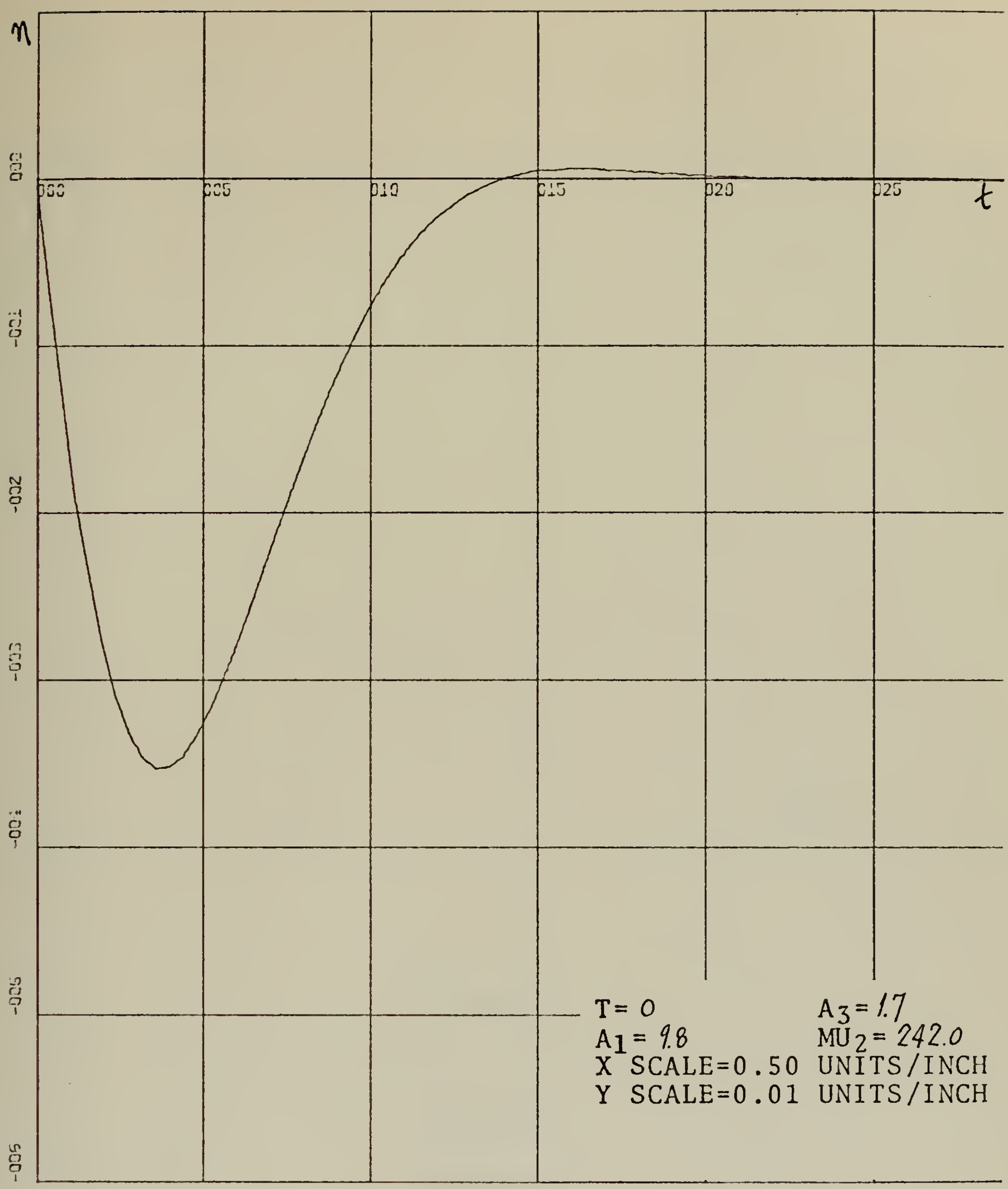


Figure 12. Transient response of a hypothetical system with zero time delay.

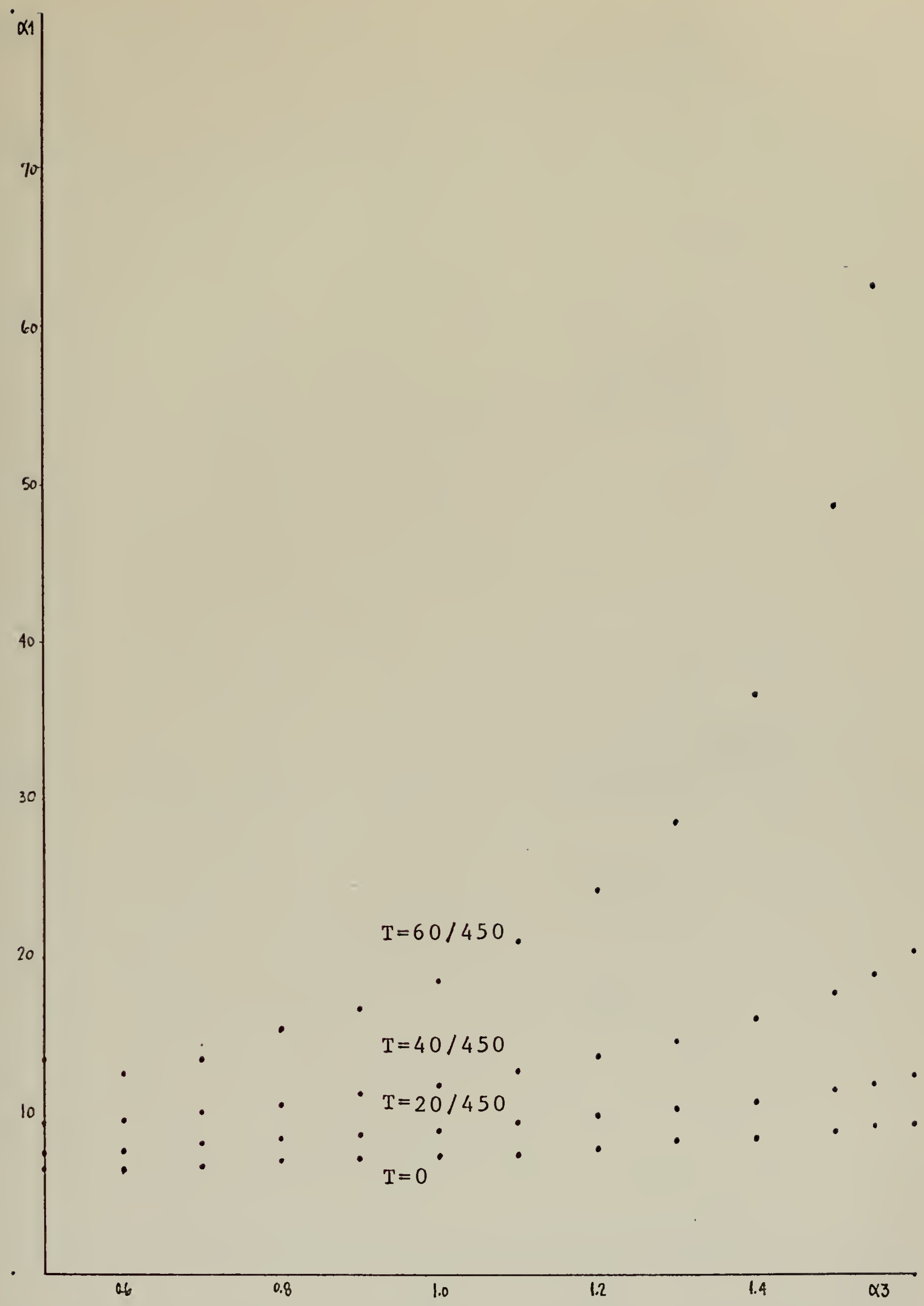


Figure 13. Projection in the α_3, α_1 parameter plane of the points of the parameter space that guarantee that each of the systems will have a pair of roots at $-3 \pm j3$.

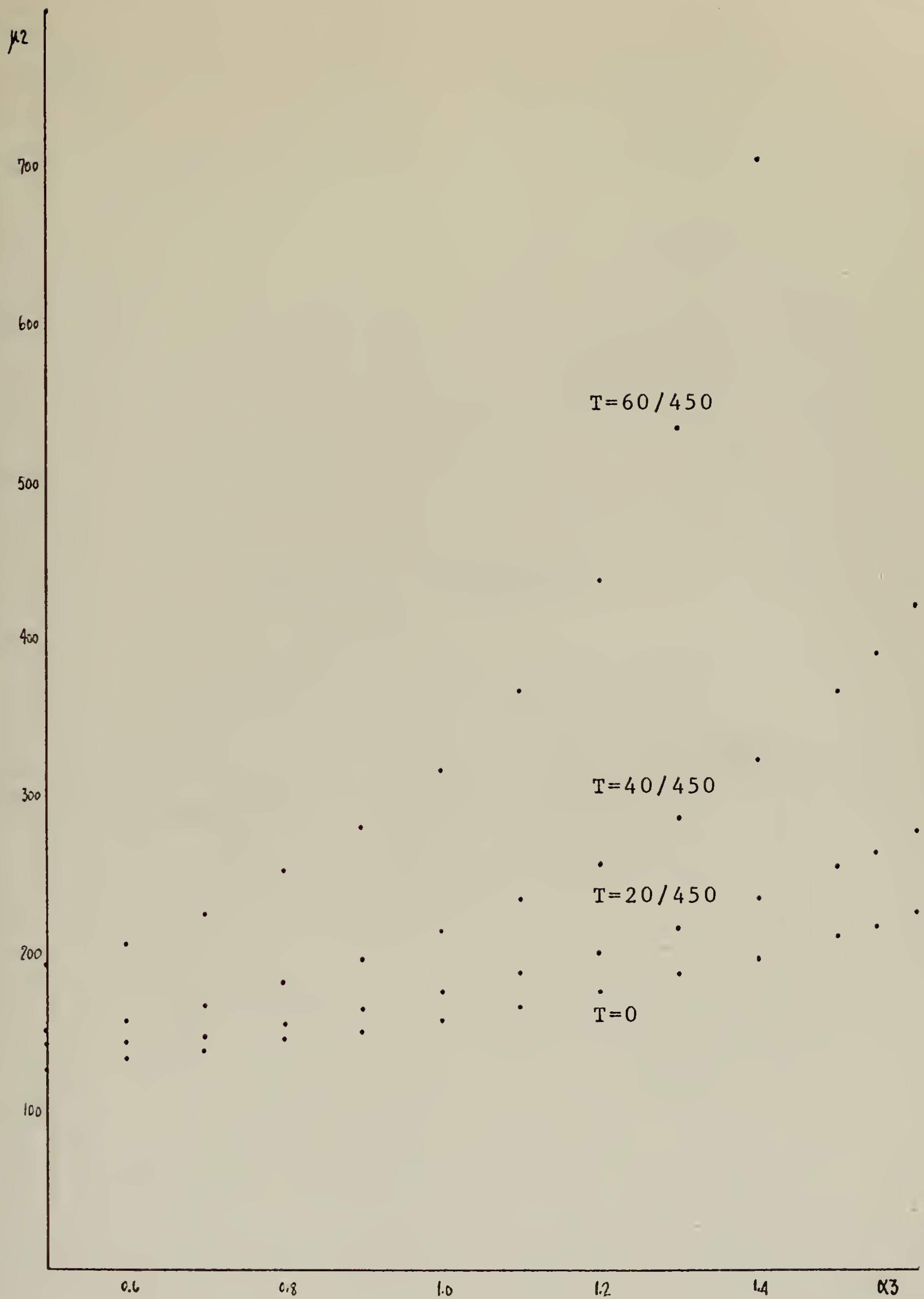


Figure 14. Projection in the α_3, μ_2 parameter plane of the points of the parameter space that guarantee that each of the systems will have a pair of roots at $-3 \pm j3$.

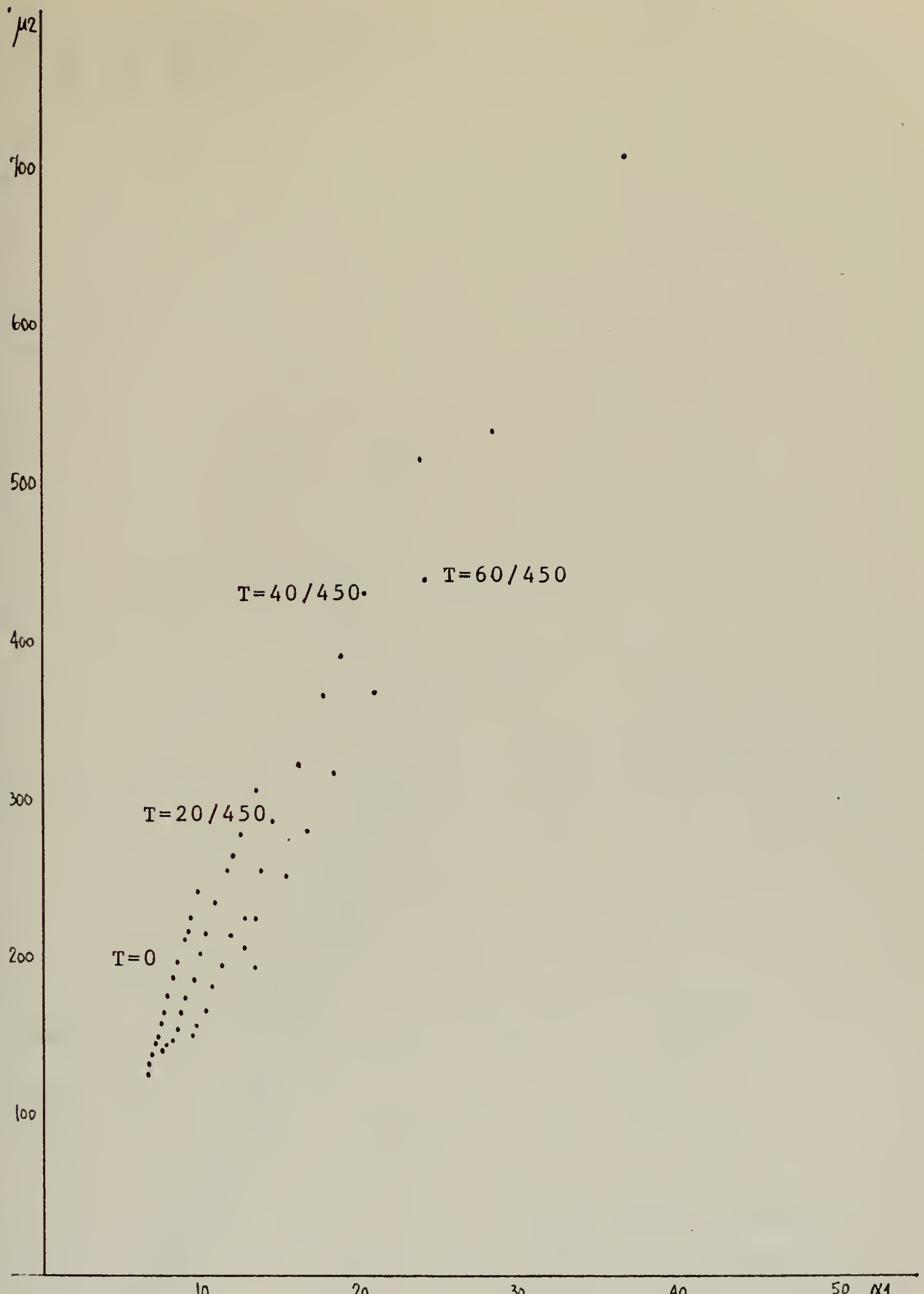
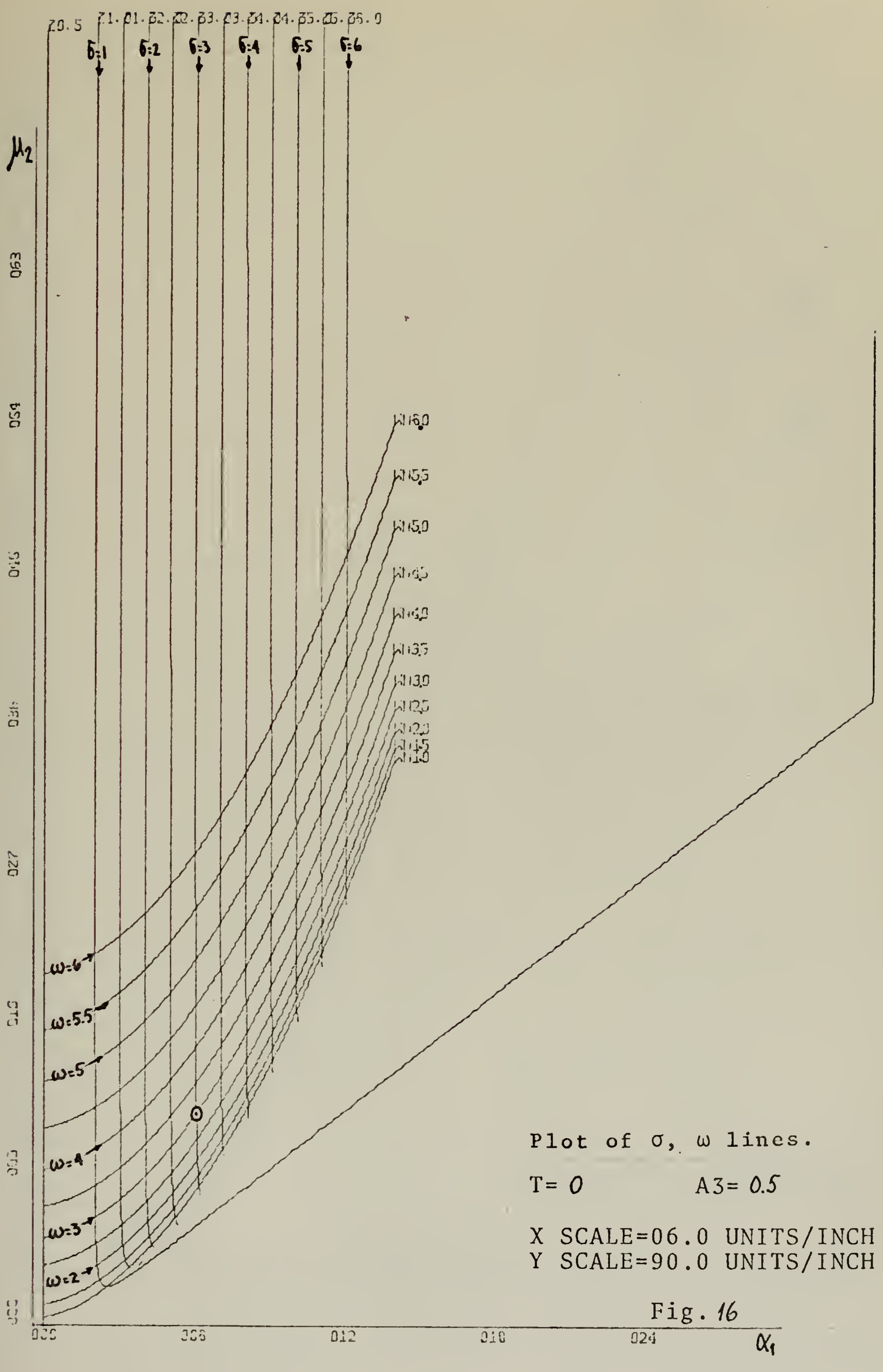


Figure 15. Projection in the α_3, μ_2 parameter plane of the points of the parameter space that guarantee that each of the systems will have a pair of roots at $-3 \pm j3$.

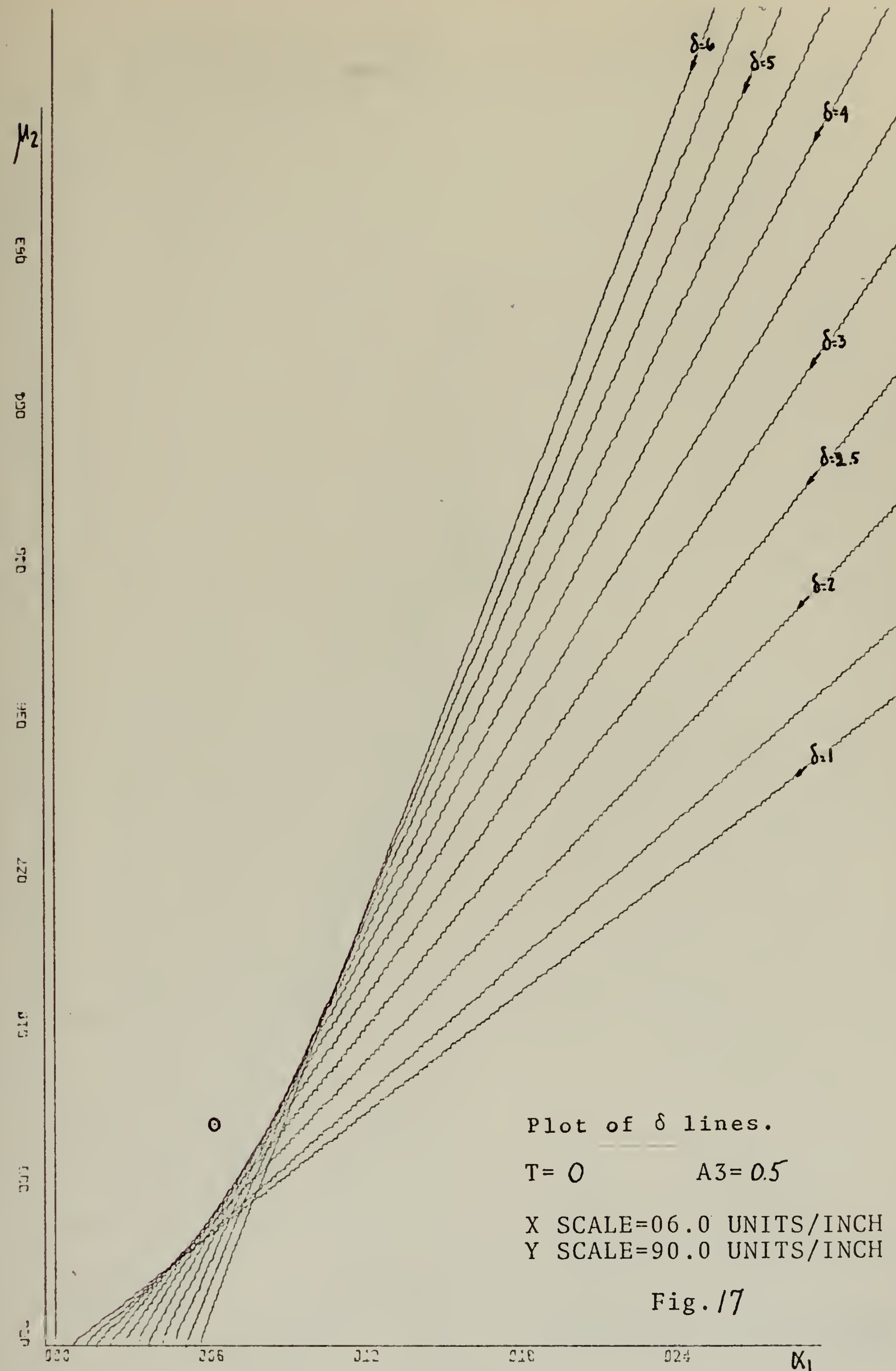


Plot of σ, ω lines.

T= 0 A3= 0.5

X SCALE=06.0 UNITS/INCH
Y SCALE=90.0 UNITS/INCH

Fig. 16



μ_2
063
054
036
027
δ=6
δ=5
δ=4
δ=3
δ=2.5
δ=2
δ=1
0
Plot of δ lines.
T= 0 A3= 0.5
X SCALE=06.0 UNITS/INCH
Y SCALE=90.0 UNITS/INCH
006
024
α_1

Fig. 17

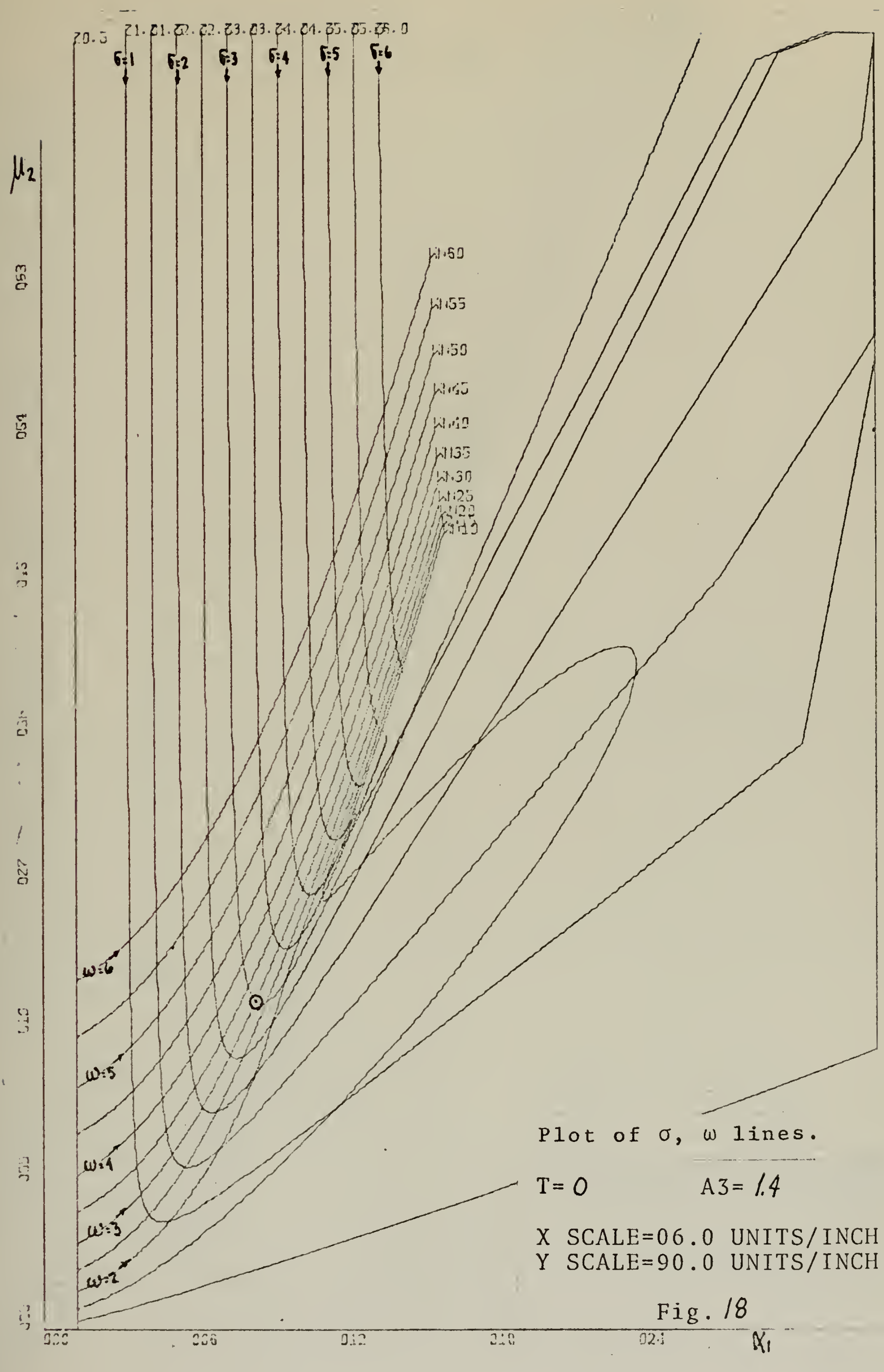

Plot of σ, ω lines.
T= 0 A3= 1.4
X SCALE=06.0 UNITS/INCH
Y SCALE=90.0 UNITS/INCH

Fig. 18

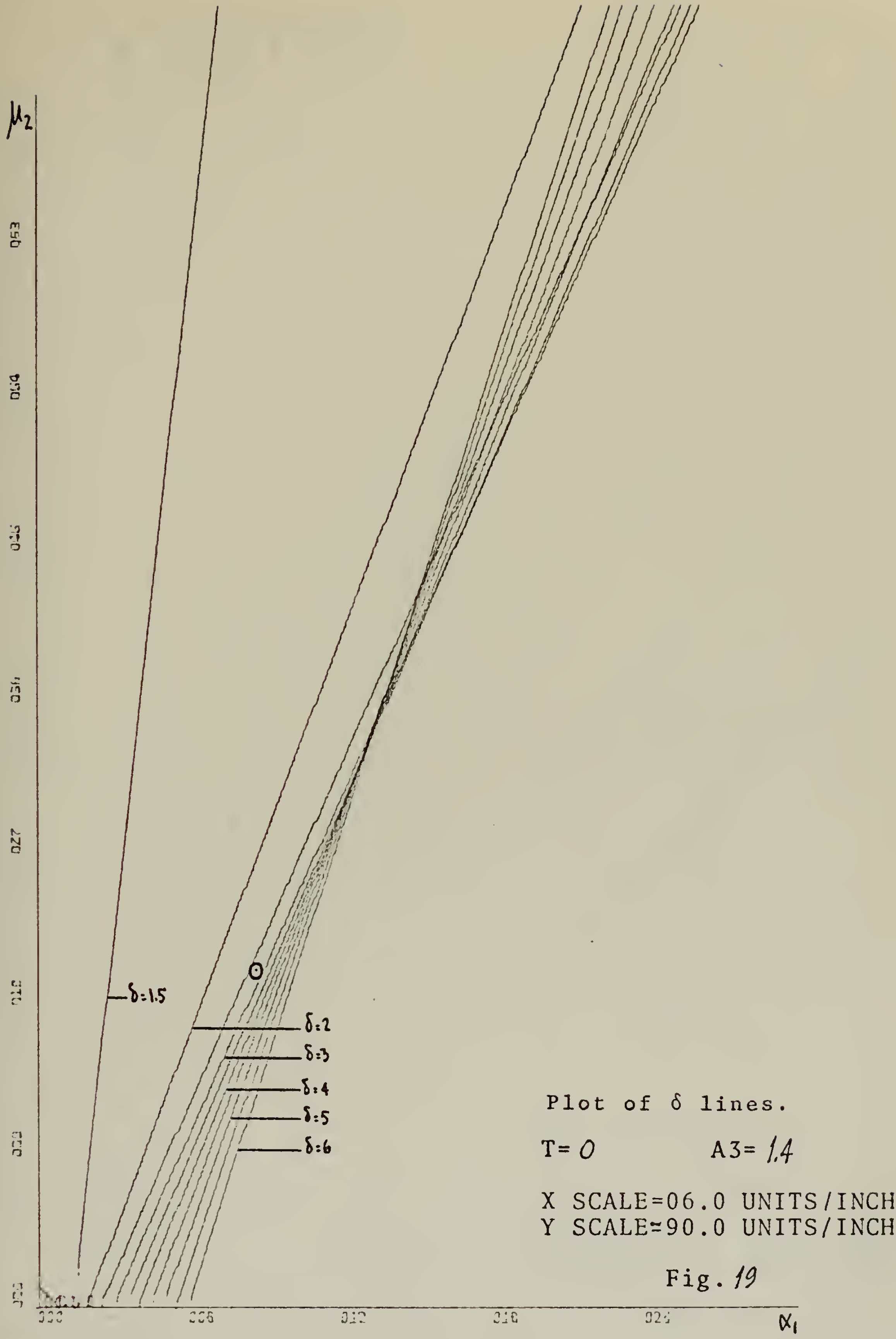

μ_2
α_1
δ=1.5
δ=2
δ=3
δ=4
δ=5
δ=6
O
Plot of δ lines.
T= 0 A3= 1.4
X SCALE=06.0 UNITS/INCH
Y SCALE=90.0 UNITS/INCH

Fig. 19

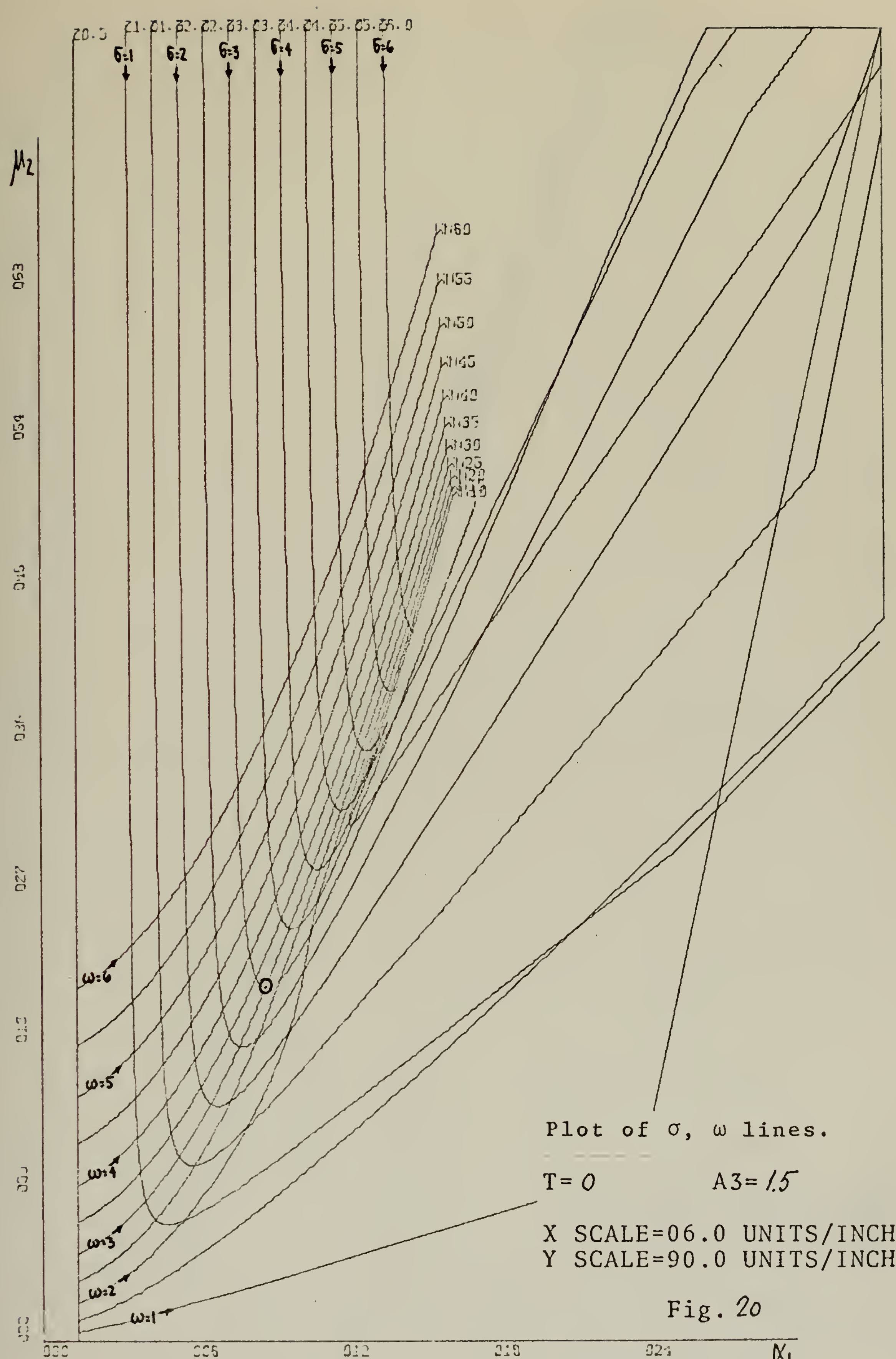
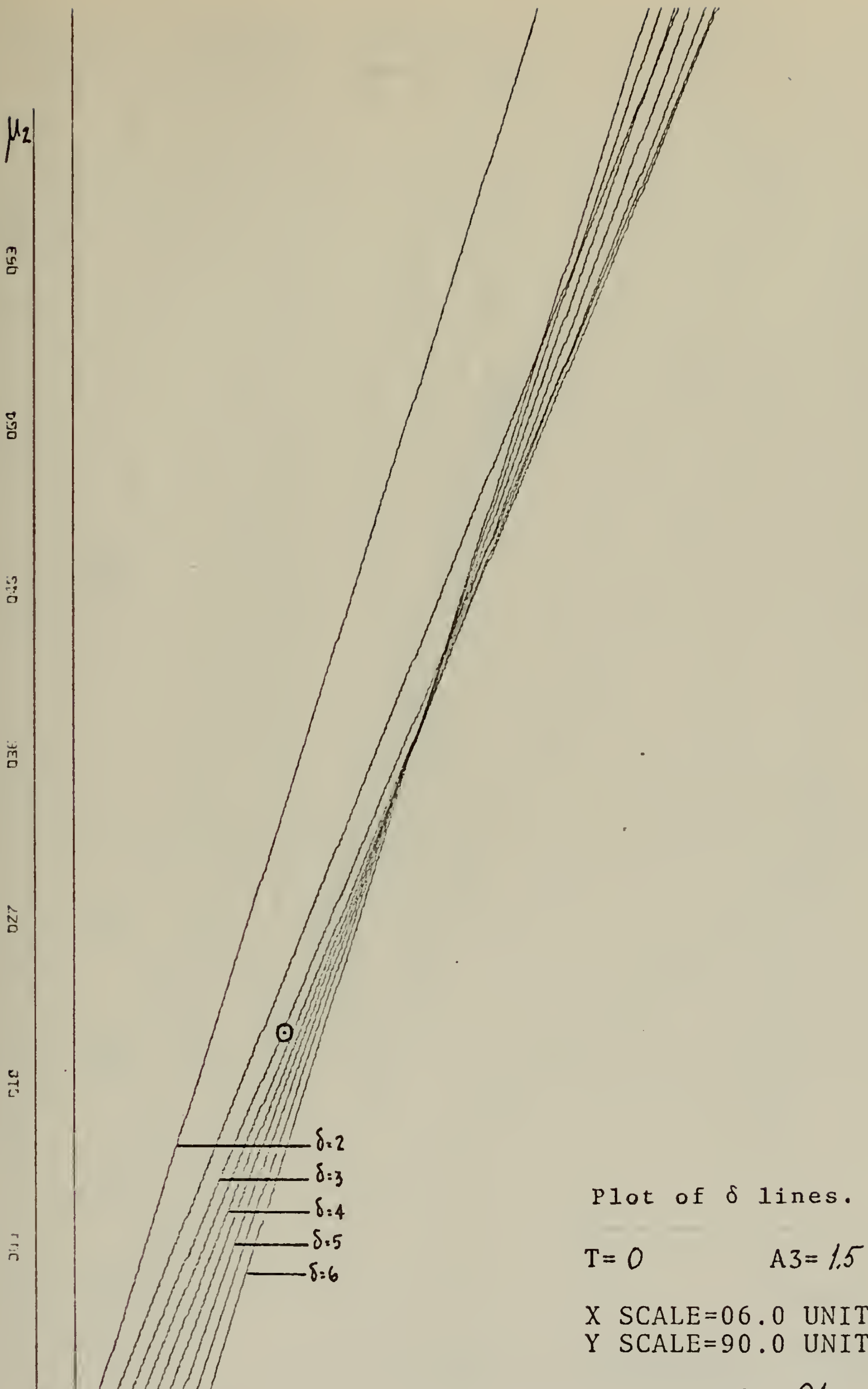

σ=1
σ=2
σ=3
σ=4
σ=5
σ=6
ω=1
ω=2
ω=3
ω=4
ω=5
ω=6
μ_2
α_1
Plot of σ, ω lines.
T= 0 A3= 1.5
X SCALE=06.0 UNITS/INCH
Y SCALE=90.0 UNITS/INCH

Fig. 20

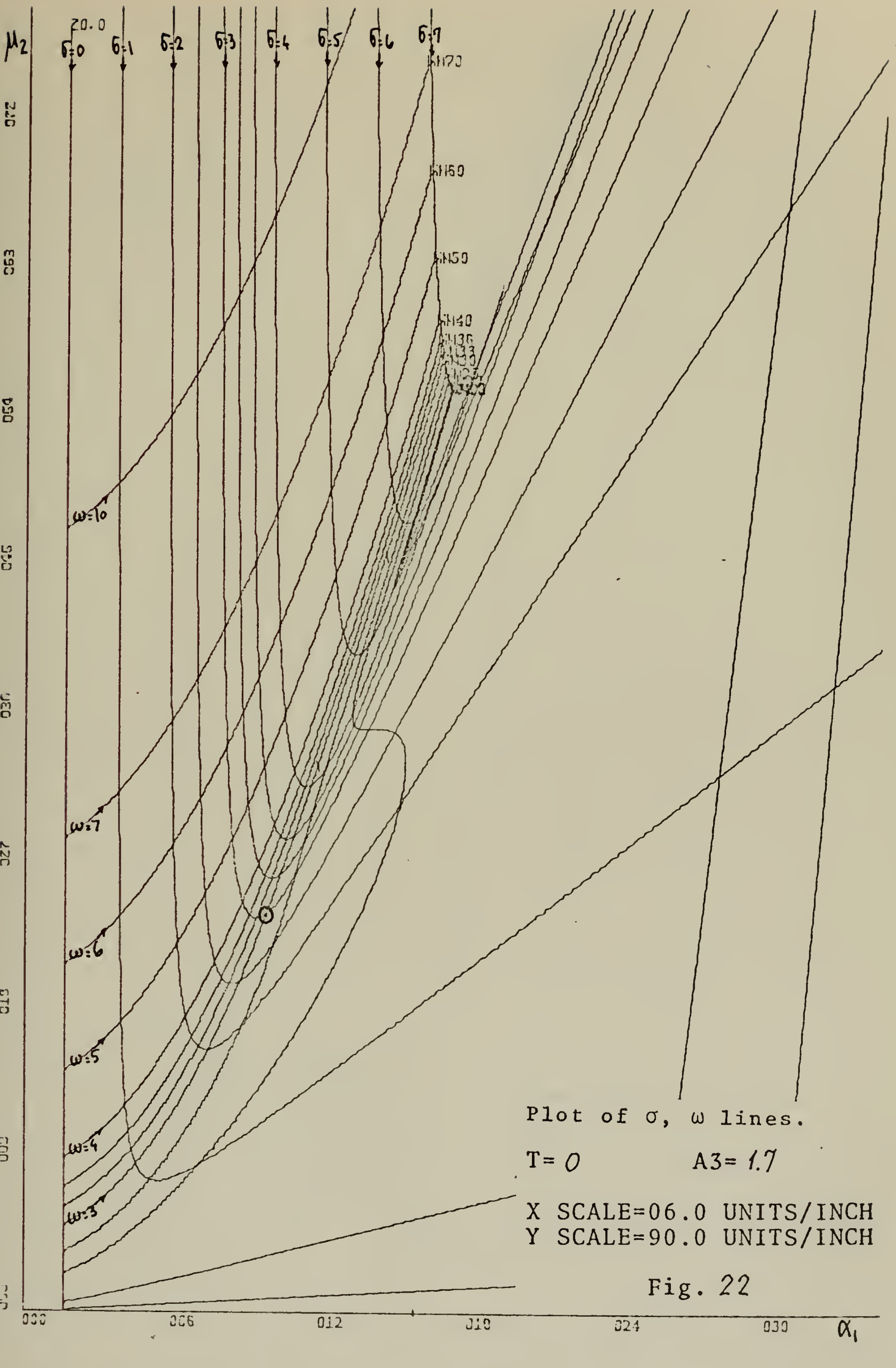


Plot of δ lines.

T= 0 A3= 1.5

X SCALE=06.0 UNITS/INCH
Y SCALE=90.0 UNITS/INCH

Fig. 21

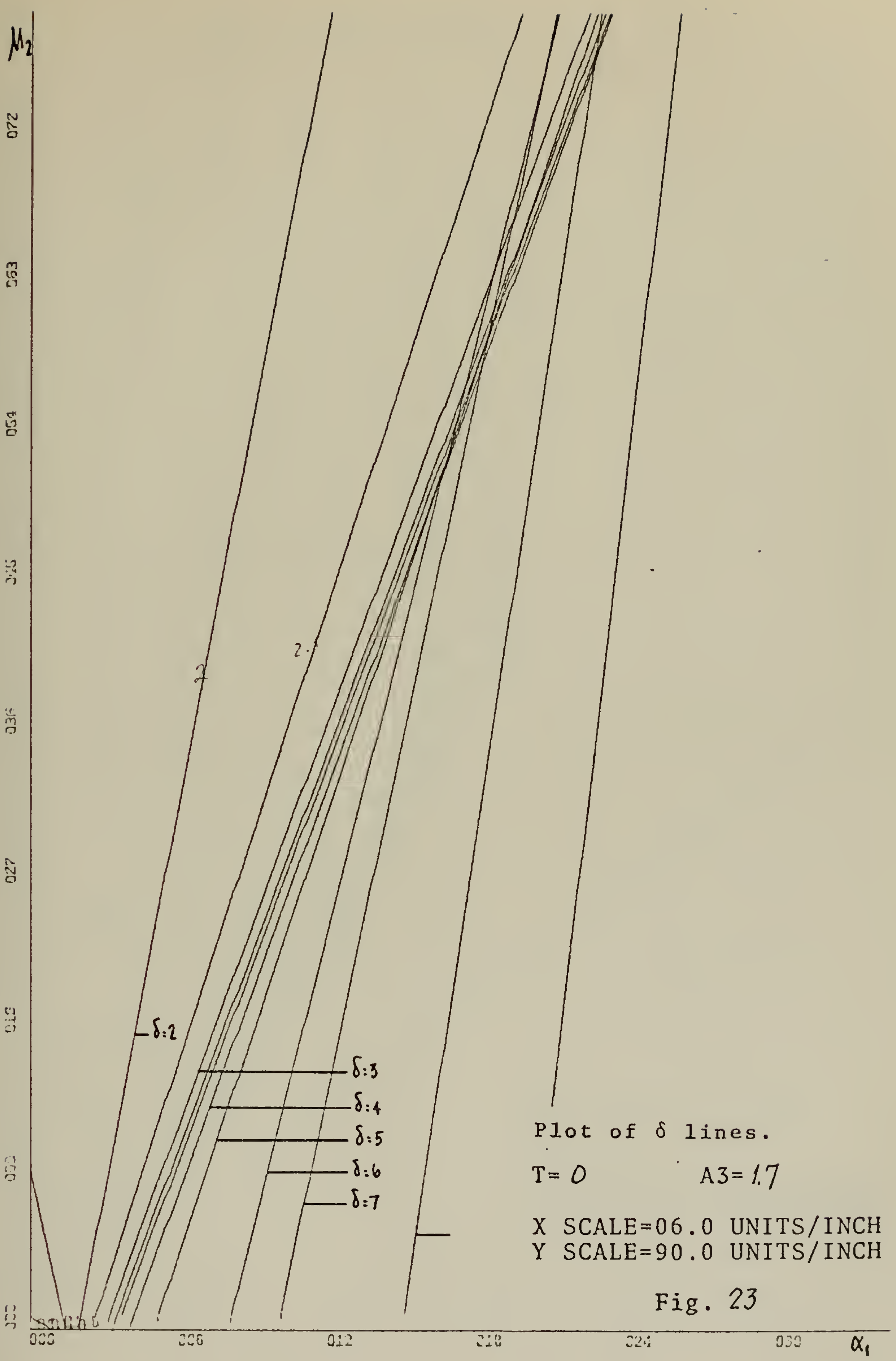


Plot of σ, ω lines.

T= 0 A3= 1.7

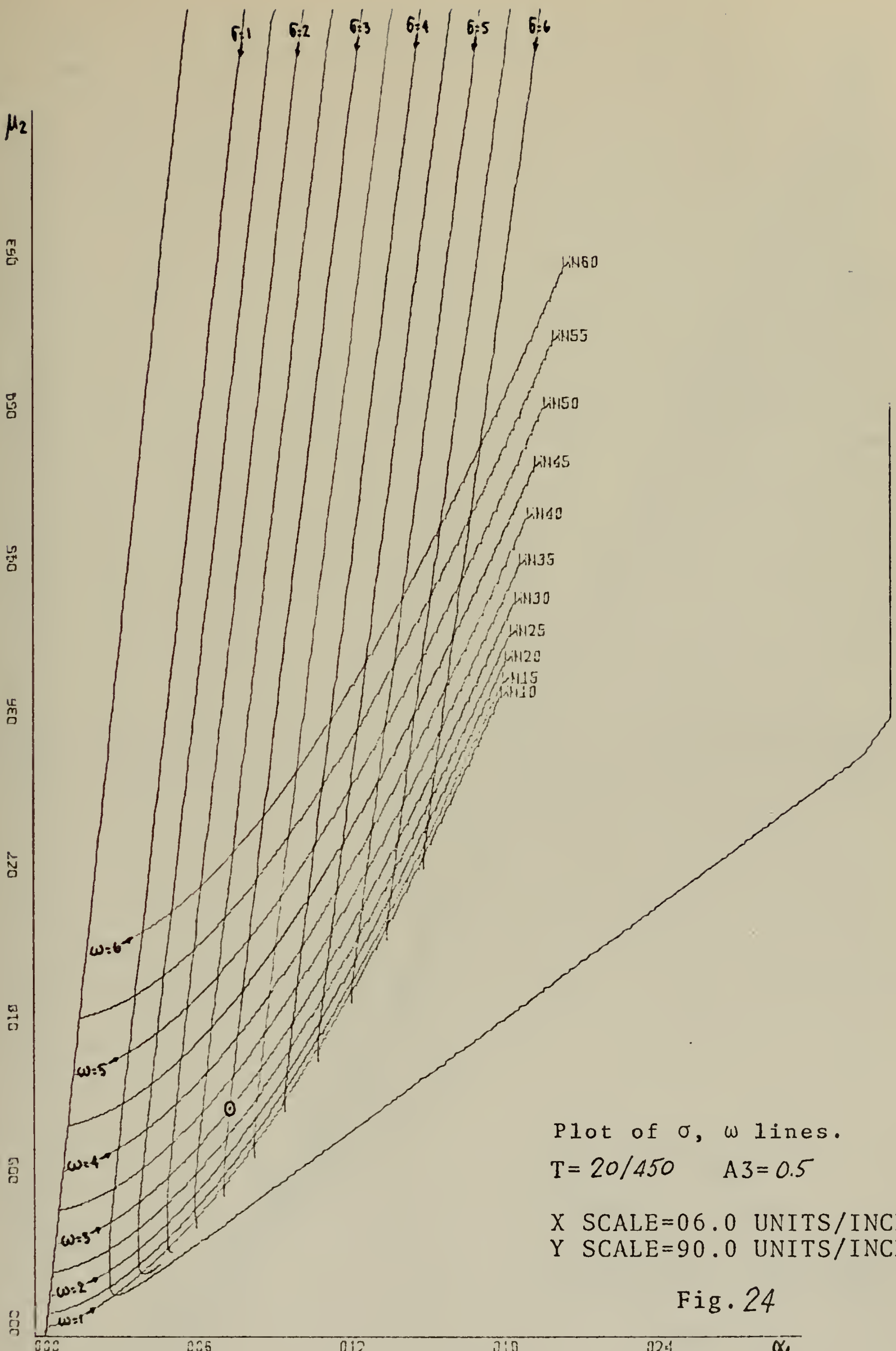
X SCALE=06.0 UNITS/INCH
Y SCALE=90.0 UNITS/INCH

Fig. 22



μ_2
072
063
027
δ:2
δ:3
δ:4
δ:5
δ:6
δ:7
Plot of δ lines.
T= 0
A3= 1.7
X SCALE=06.0 UNITS/INCH
Y SCALE=90.0 UNITS/INCH
012
α_1

Fig. 23

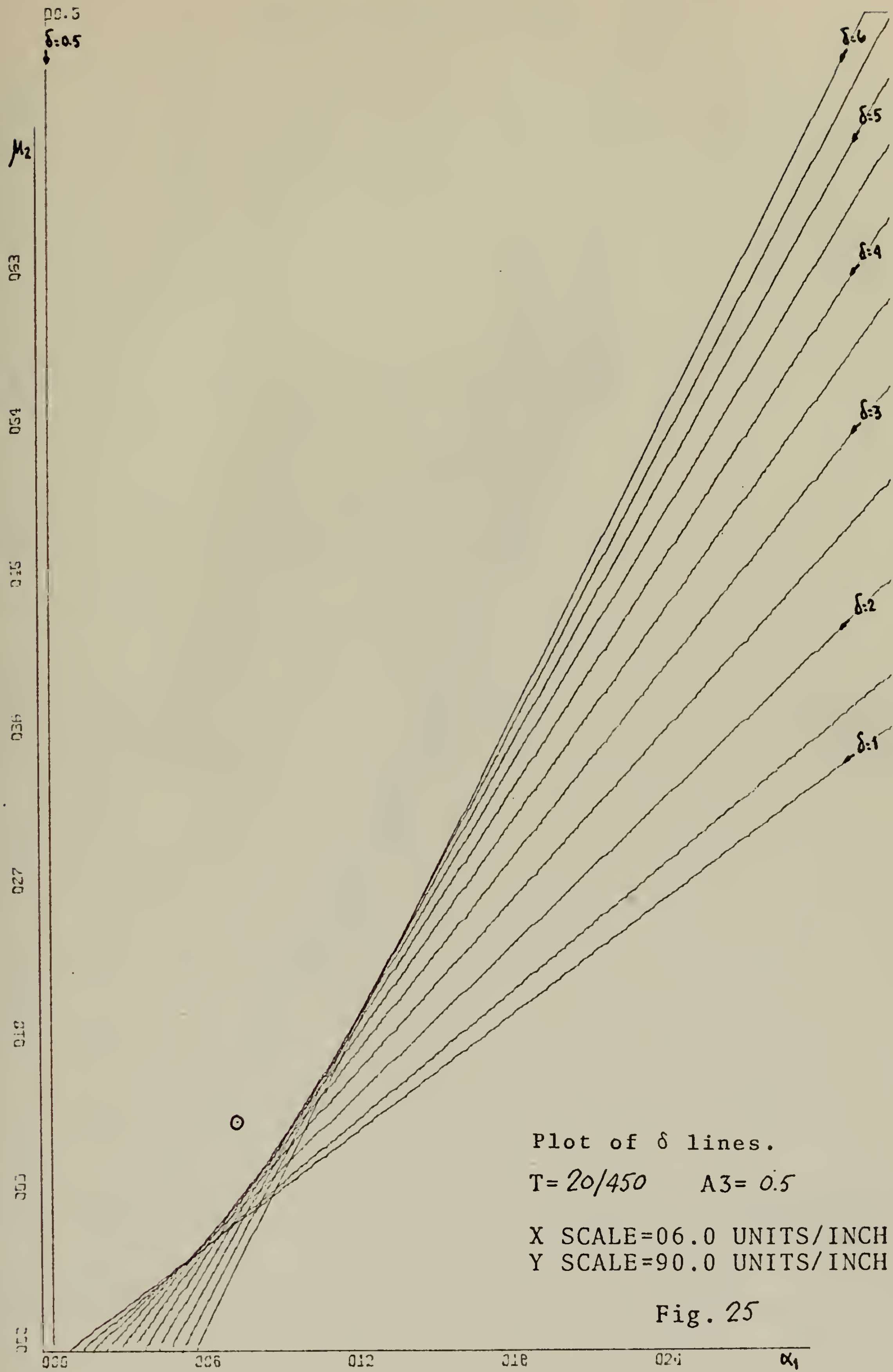


Plot of σ, ω lines.

T= 20/450   A3= 0.5

X SCALE=06.0 UNITS/INCH
Y SCALE=90.0 UNITS/INCH

Fig. 24

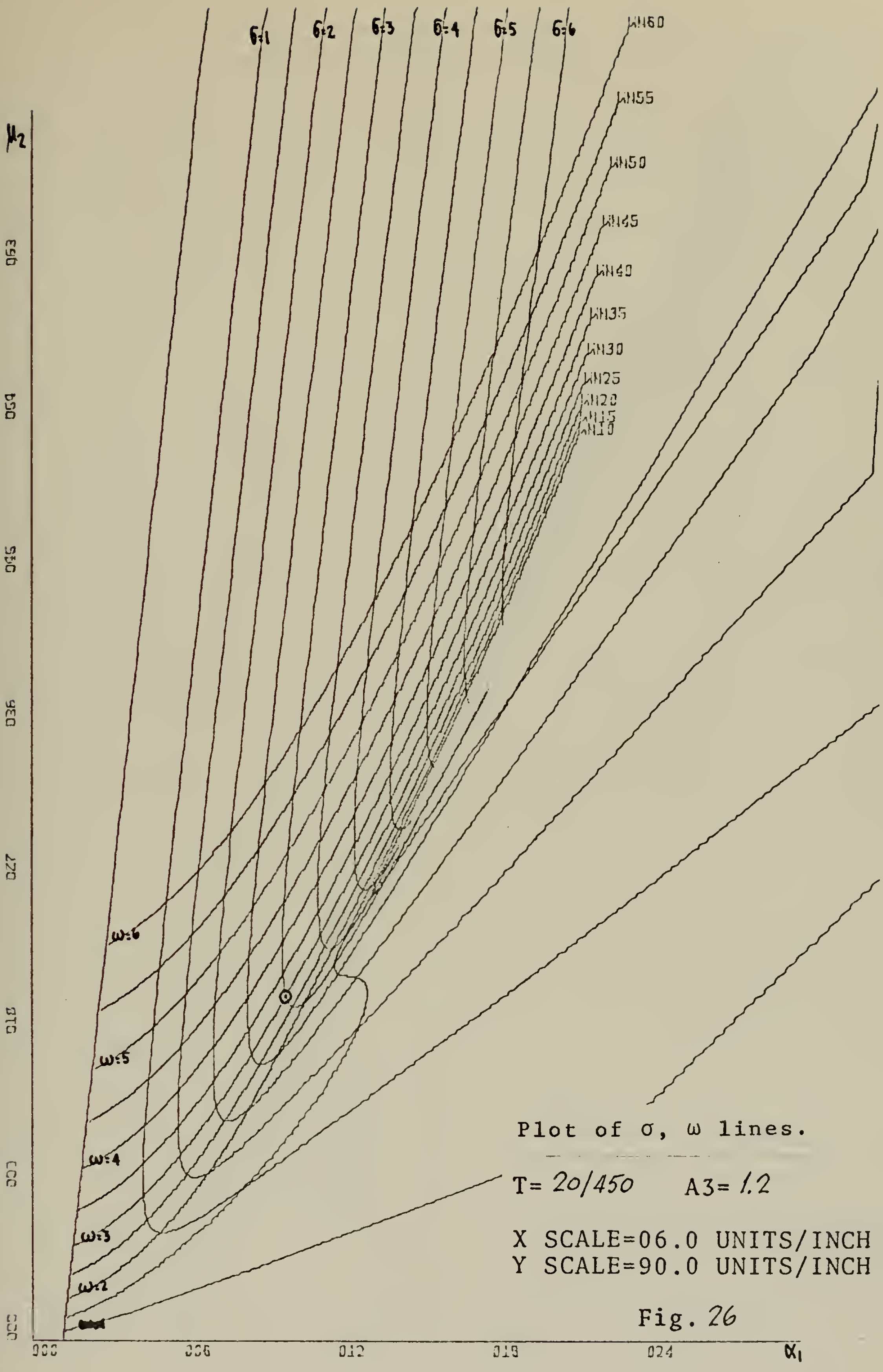


Plot of δ lines.

T= 20/450 A3= 0.5

X SCALE=06.0 UNITS/INCH
Y SCALE=90.0 UNITS/INCH

Fig. 25

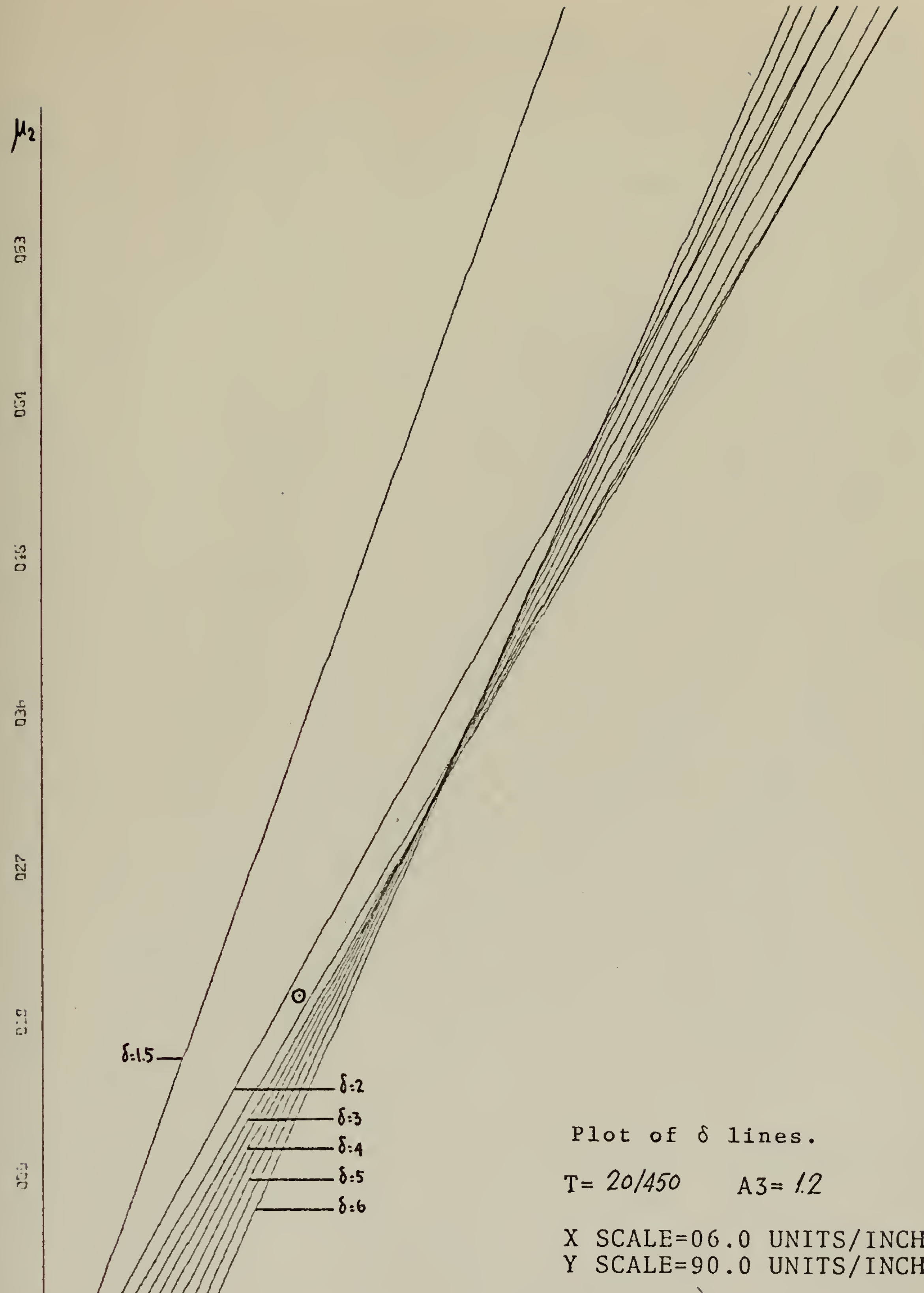


Plot of σ, ω lines.

T= 20/450 A3= 1.2

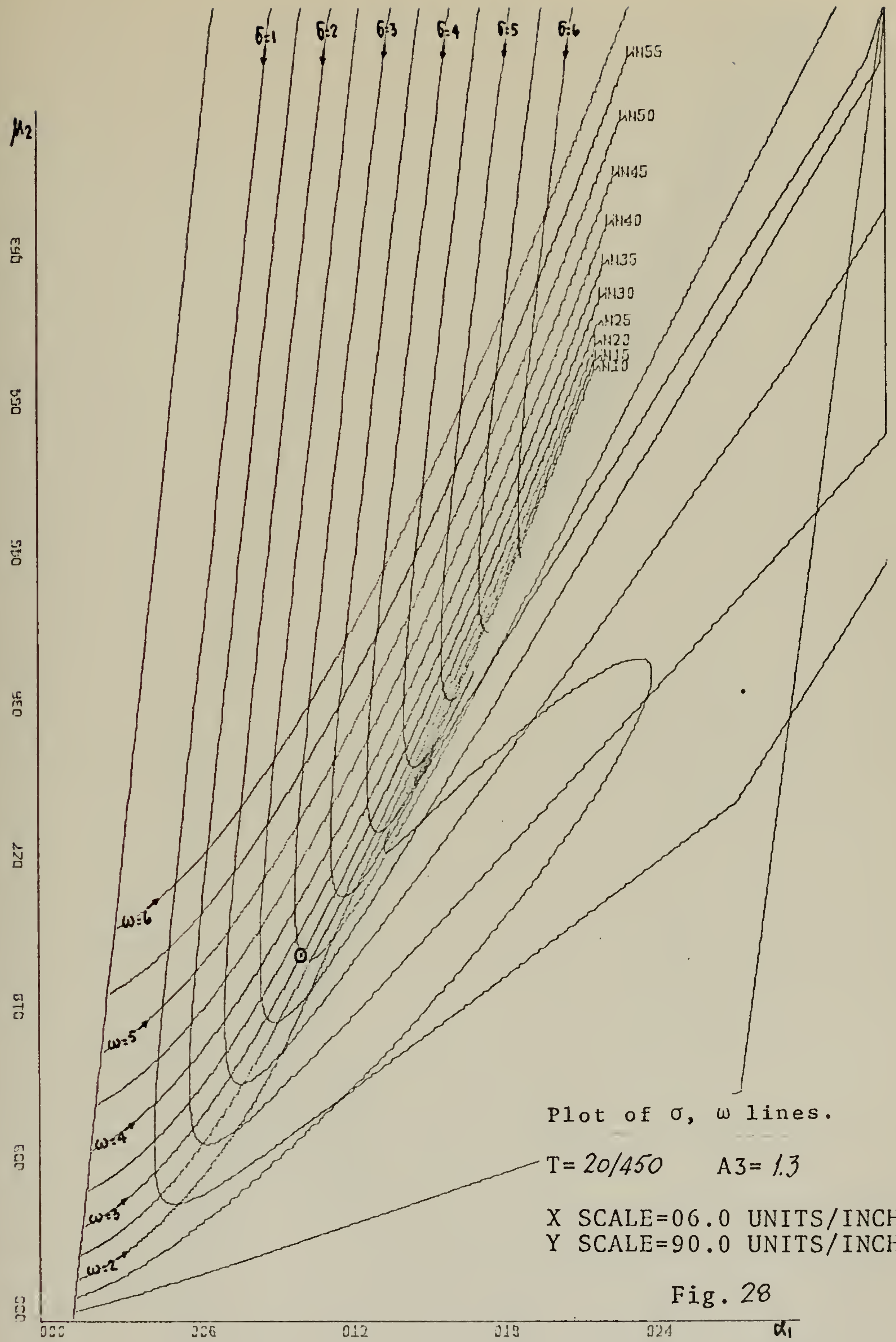
X SCALE=06.0 UNITS/INCH
Y SCALE=90.0 UNITS/INCH

Fig. 26



μ_2
α_1
δ:1.5
δ:2
δ:3
δ:4
δ:5
δ:6
Plot of δ lines.
T= 20/450 A3= 1.2
X SCALE=06.0 UNITS/INCH
Y SCALE=90.0 UNITS/INCH

Fig. 27

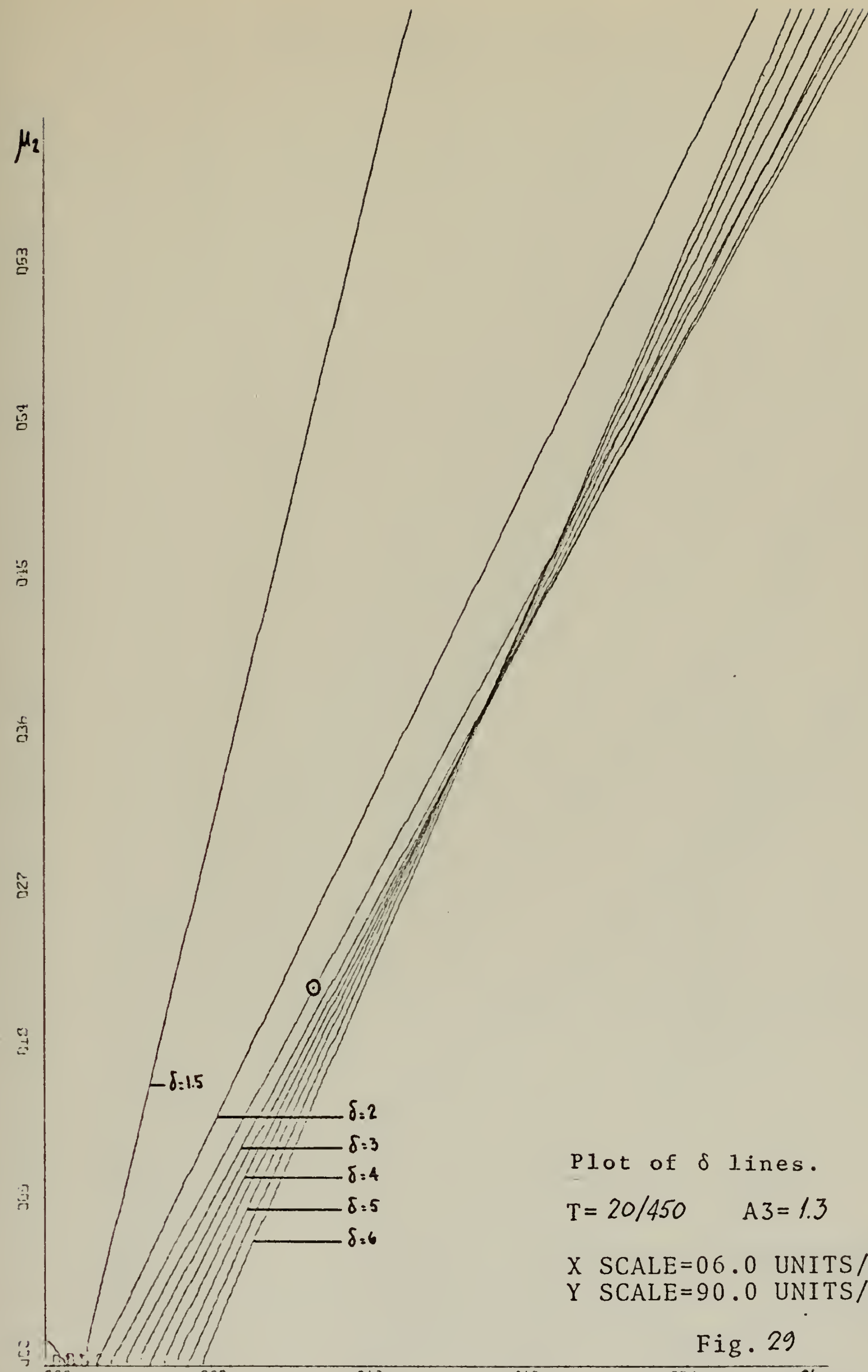


Plot of σ, ω lines.

T= 20/450 A3= 1.3

X SCALE=06.0 UNITS/INCH
Y SCALE=90.0 UNITS/INCH

Fig. 28

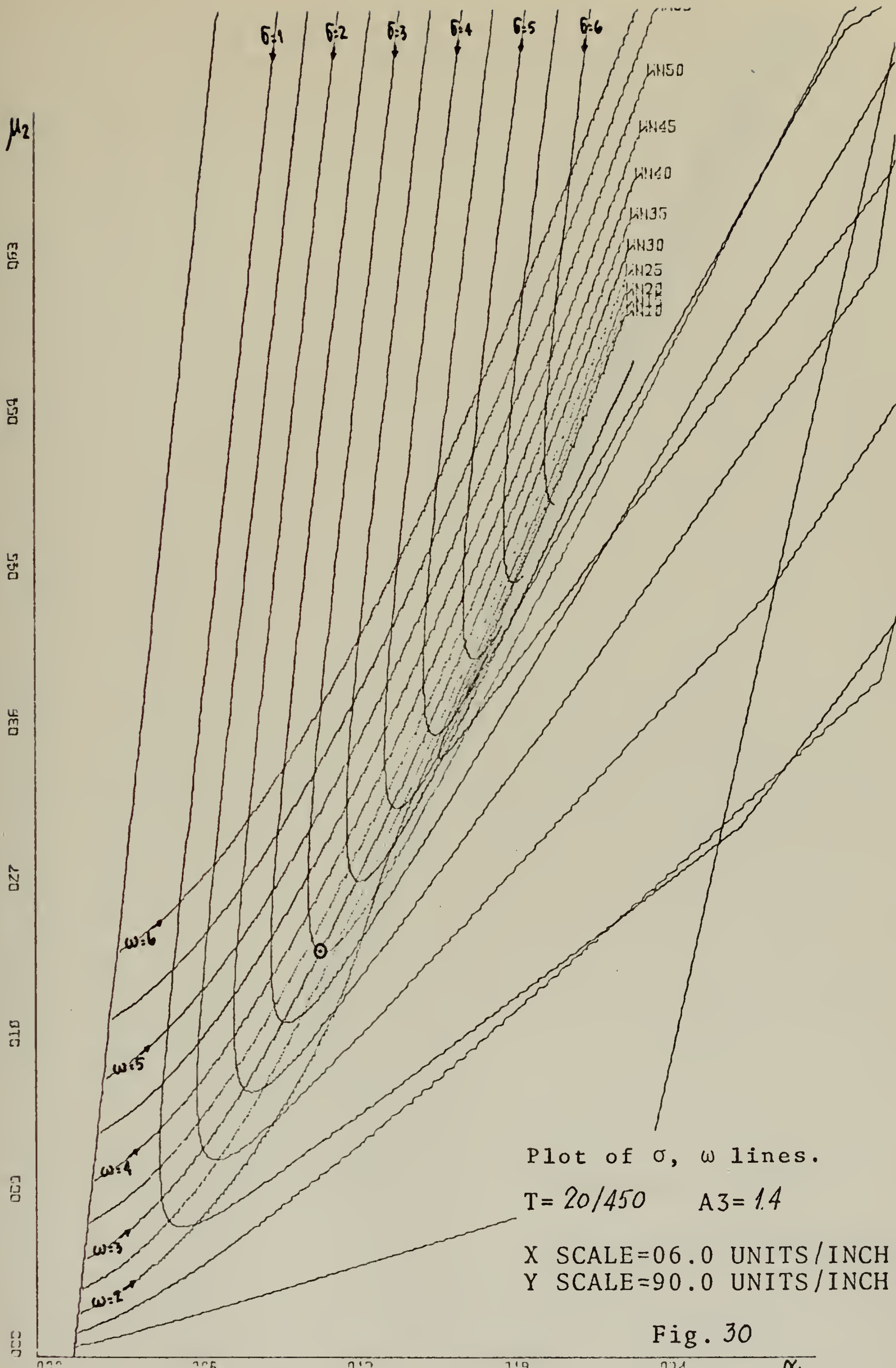


Plot of δ lines.

T= 20/450 A3= 1.3

X SCALE=06.0 UNITS/INCH
Y SCALE=90.0 UNITS/INCH

Fig. 29

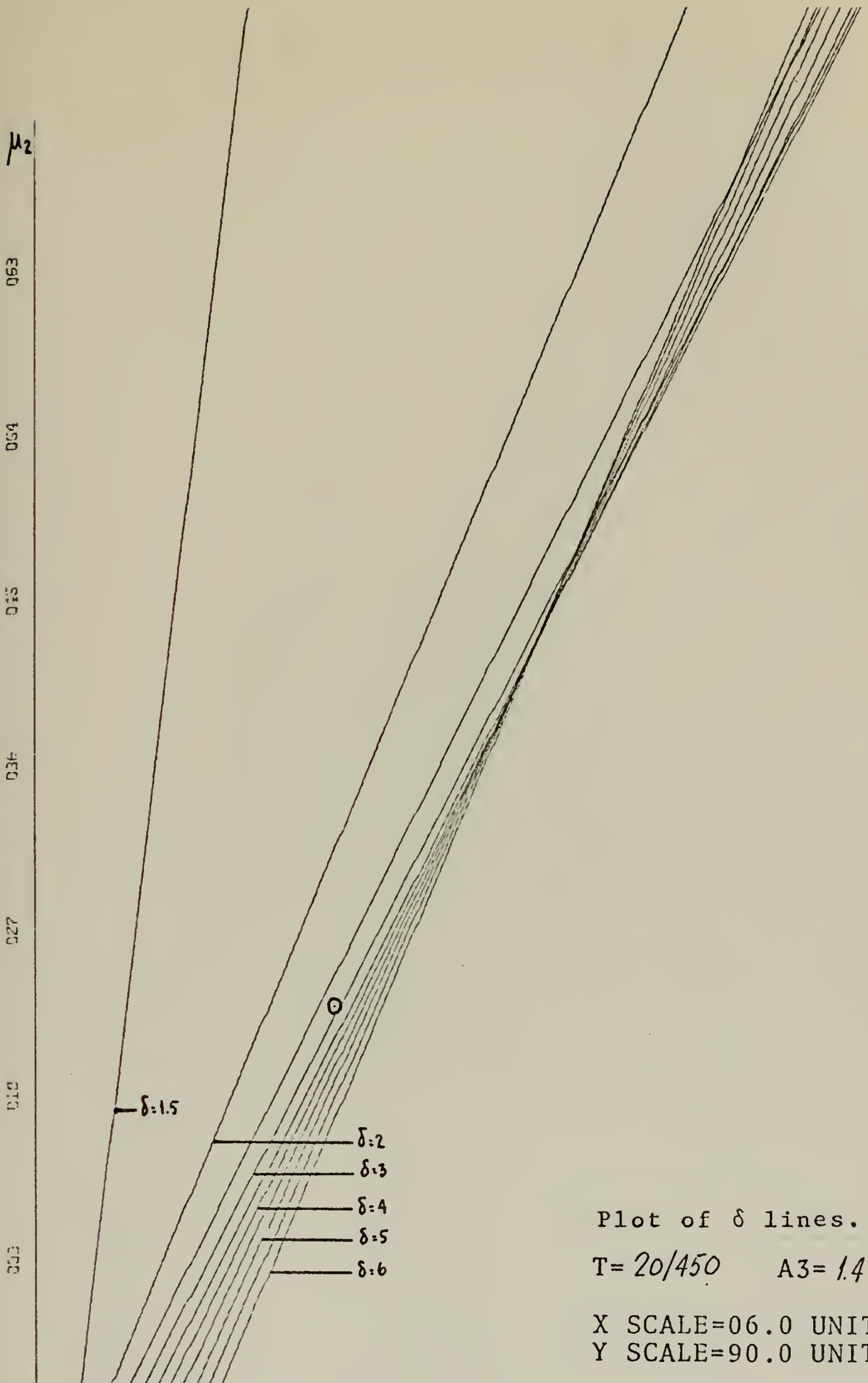


Plot of σ, ω lines.

T= 20/450   A3= 1.4

X SCALE=06.0 UNITS/INCH
Y SCALE=90.0 UNITS/INCH

Fig. 30

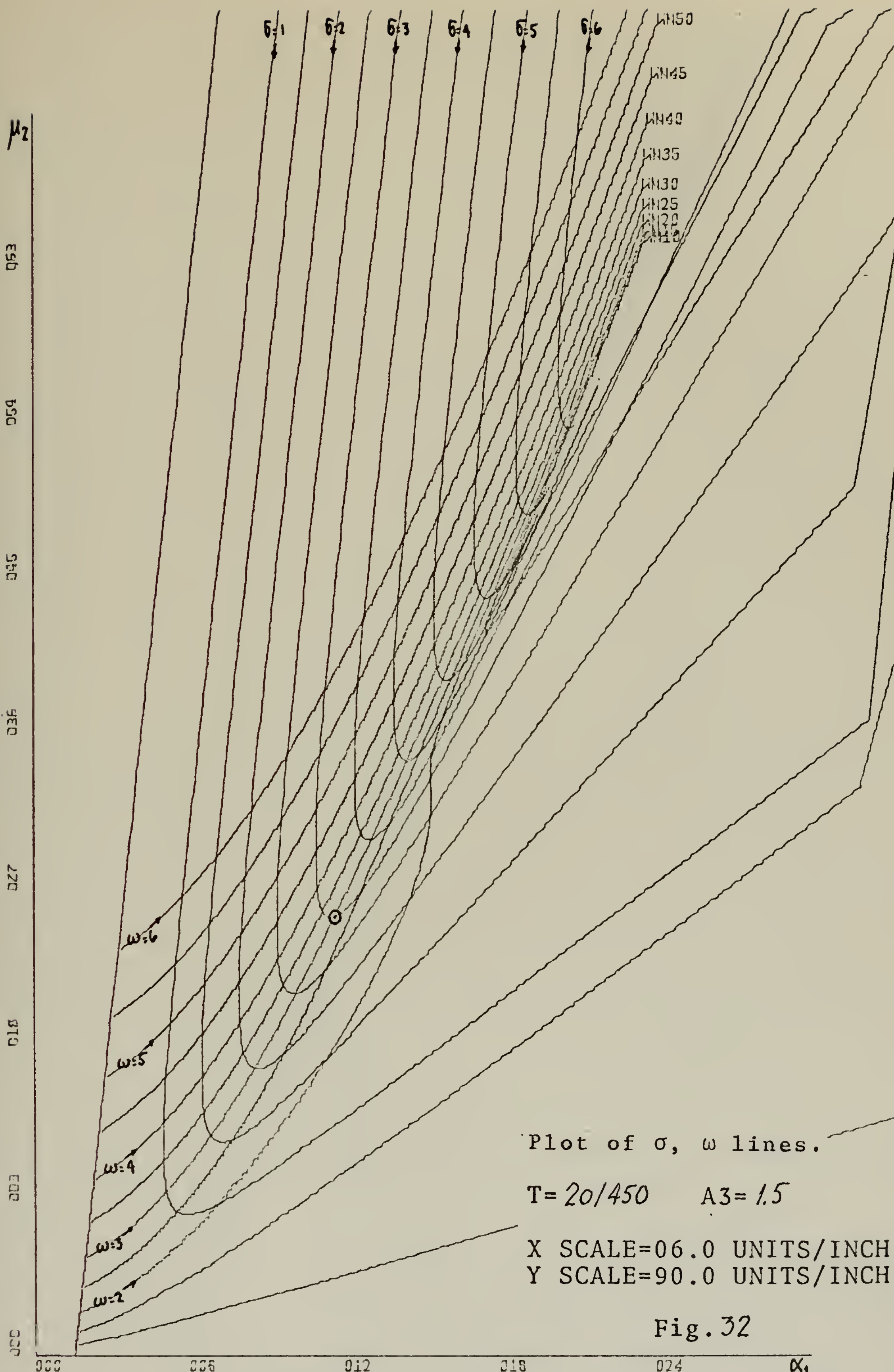


Plot of δ lines.

T= 20/450 A3= 1.4

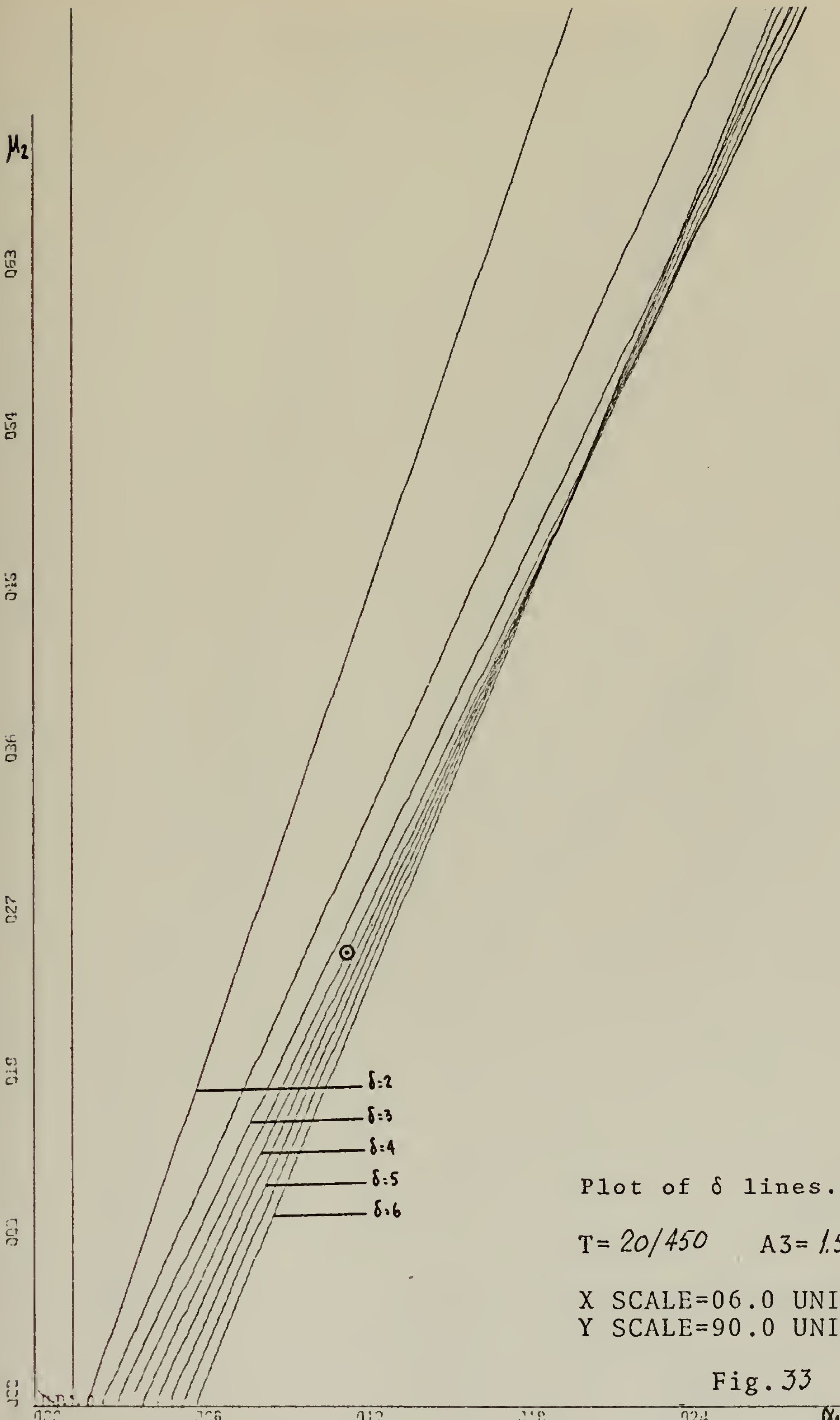
X SCALE=06.0 UNITS/INCH
Y SCALE=90.0 UNITS/INCH

Fig. 31



μ_2
σ=1
σ=2
σ=3
σ=4
σ=5
σ=6
WN50
WN45
WN40
WN35
WN30
WN25
WN20
WN10
ω=6
ω=5
ω=4
ω=3
ω=2
000
006
012
018
024
α_1
000
009
018
027
036
045
054
063
Plot of σ, ω lines.
T= 20/450 A3= 1.5
X SCALE=06.0 UNITS/INCH
Y SCALE=90.0 UNITS/INCH

Fig. 32

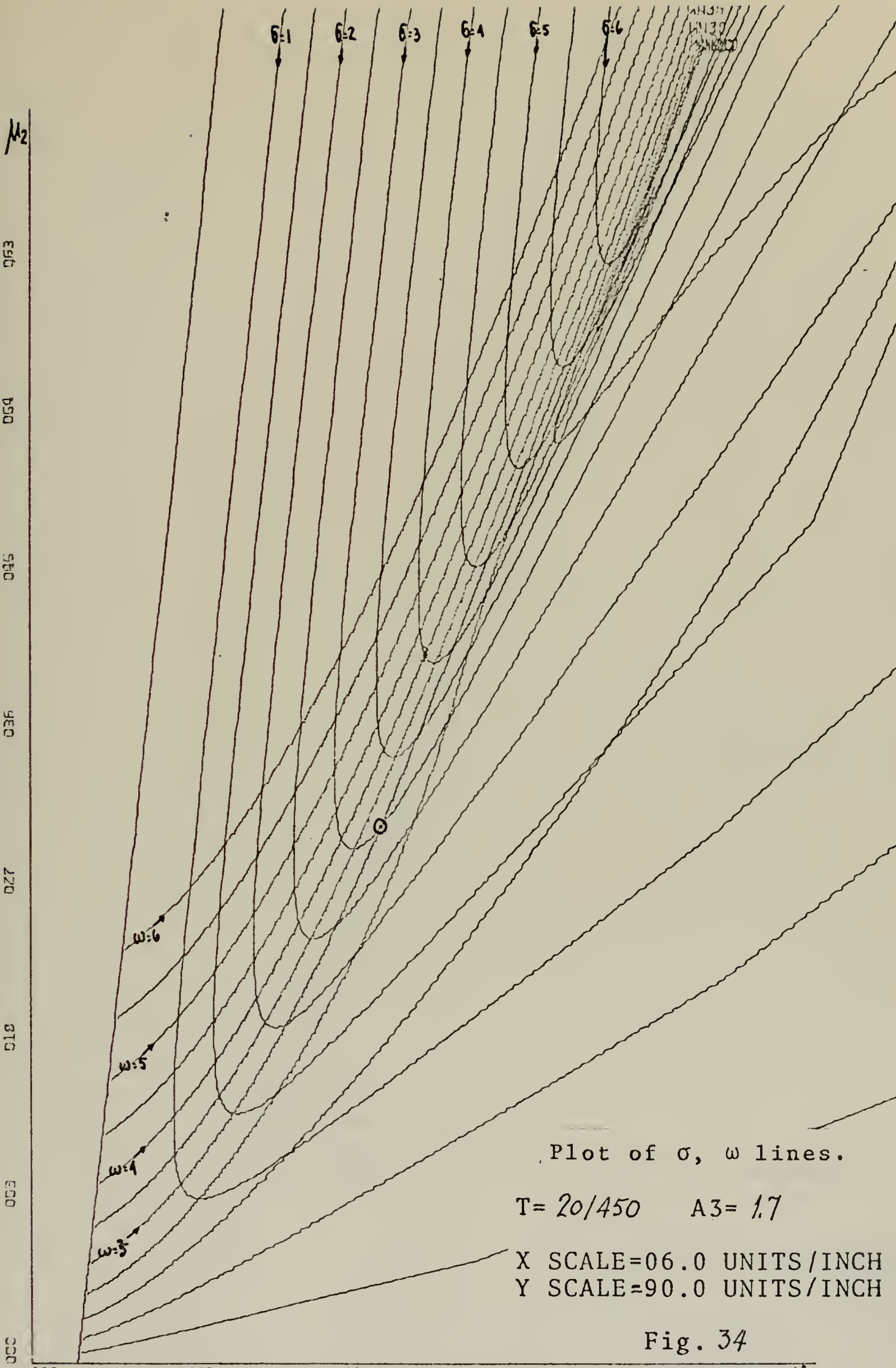


Plot of δ lines.

T= 20/450 A3= 1.5

X SCALE=06.0 UNITS/INCH
Y SCALE=90.0 UNITS/INCH

Fig. 33

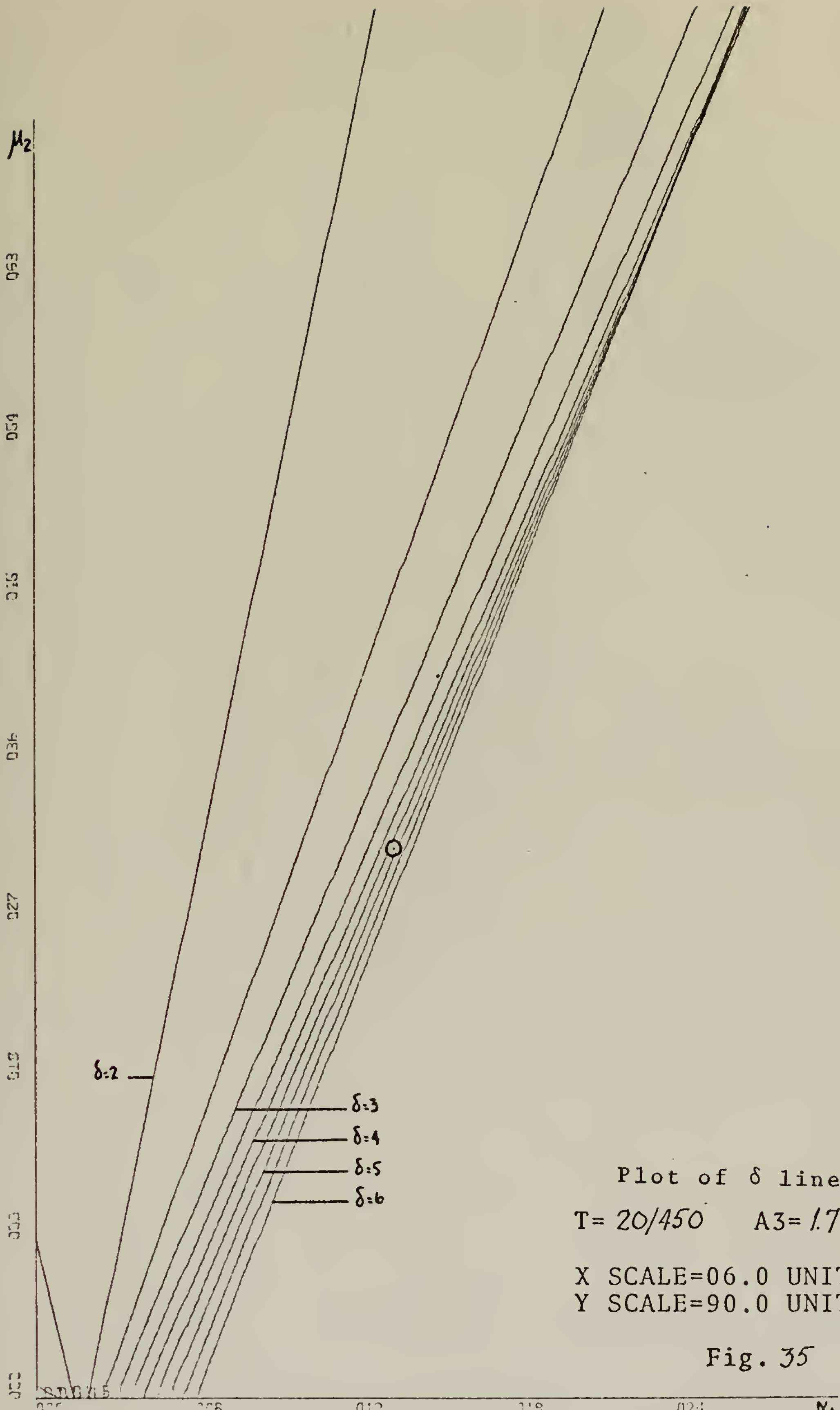


Plot of σ, ω lines.

T= 20/450 A3= 1.7

X SCALE=06.0 UNITS/INCH
Y SCALE=90.0 UNITS/INCH

Fig. 34

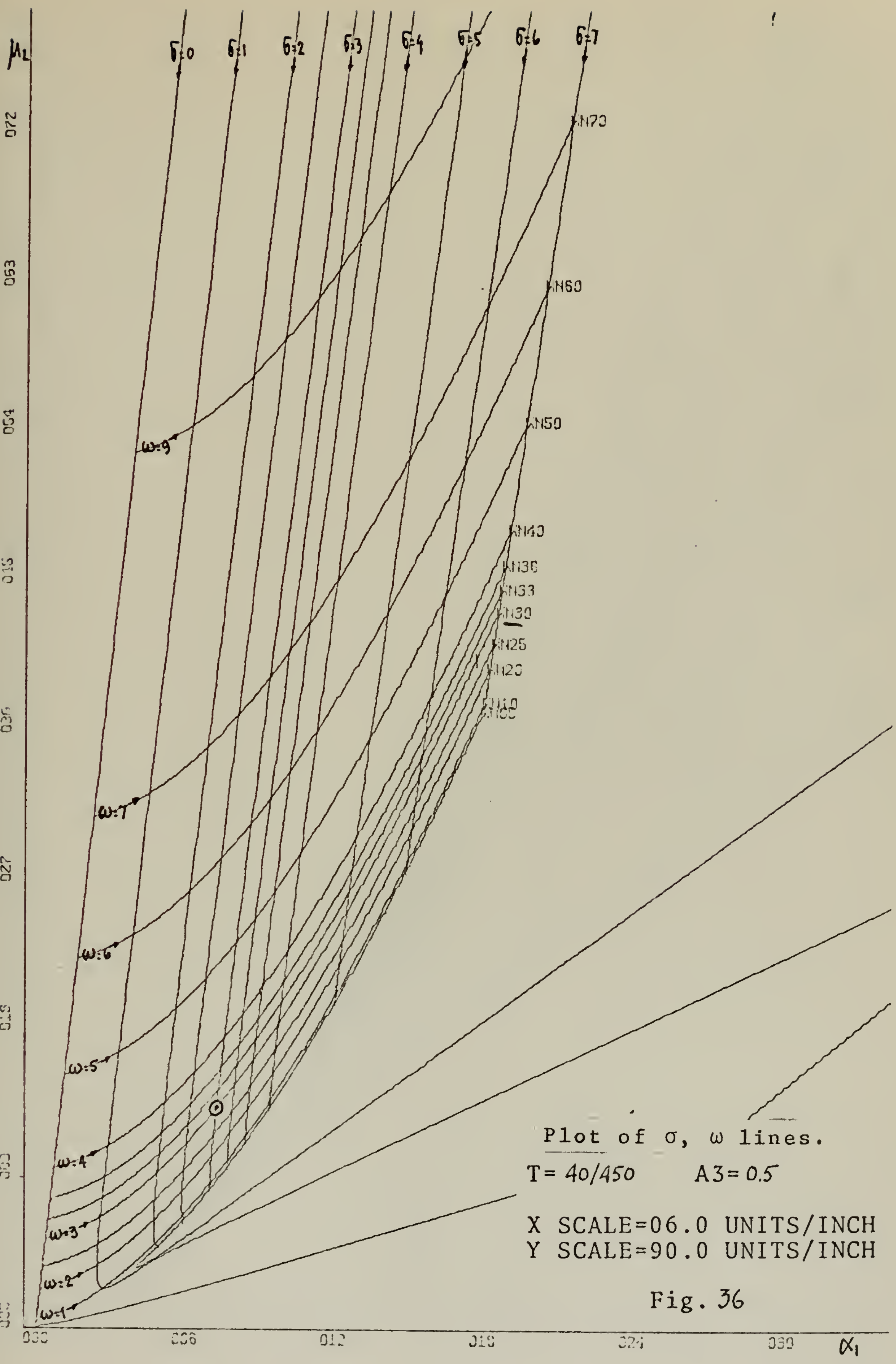


Plot of δ lines.

T= 20/450   A3= 1.7

X SCALE=06.0 UNITS/INCH
Y SCALE=90.0 UNITS/INCH

Fig. 35

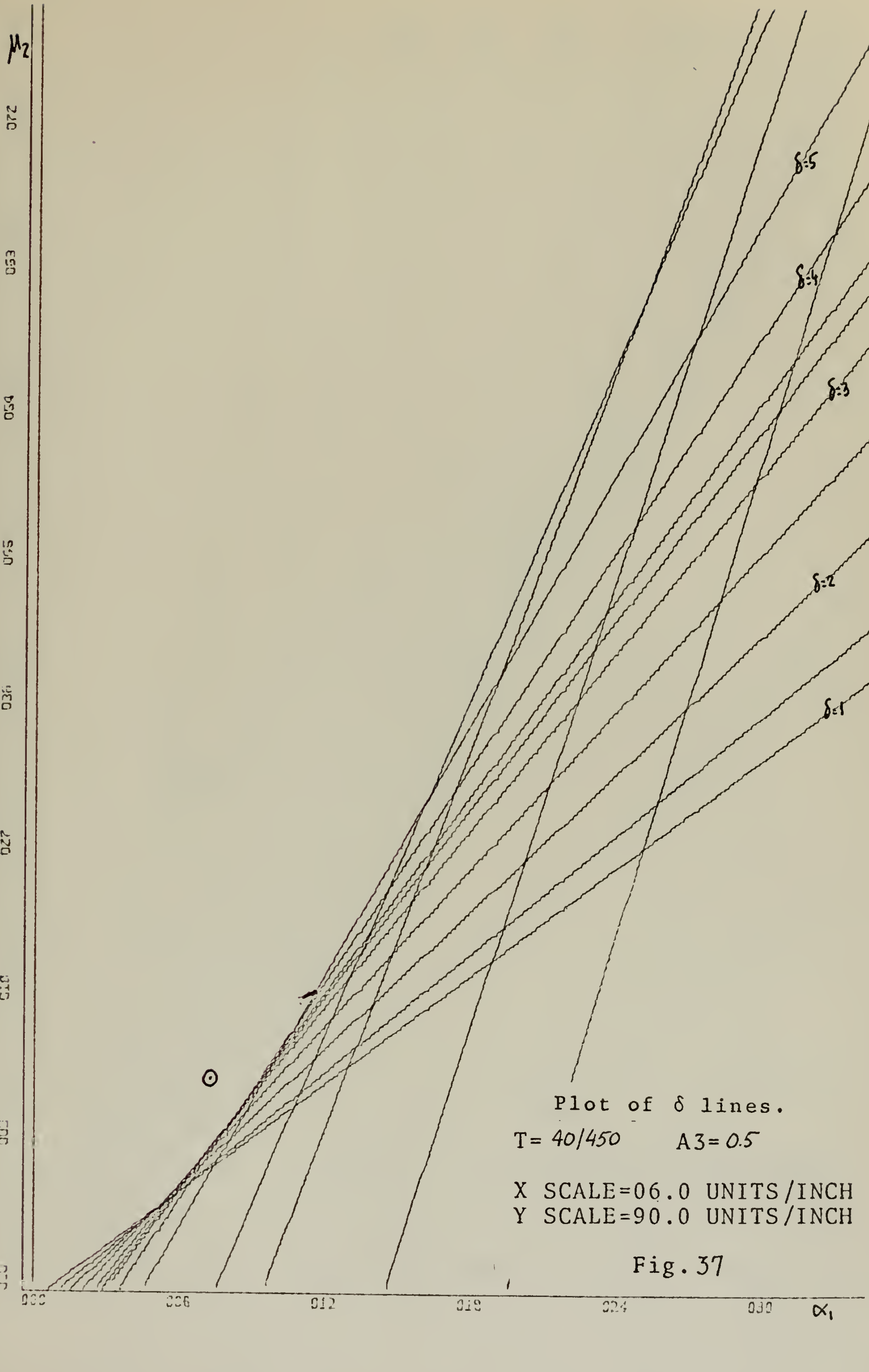


Plot of σ, ω lines.

T= 40/450 A3= 0.5

X SCALE=06.0 UNITS/INCH
Y SCALE=90.0 UNITS/INCH

Fig. 36

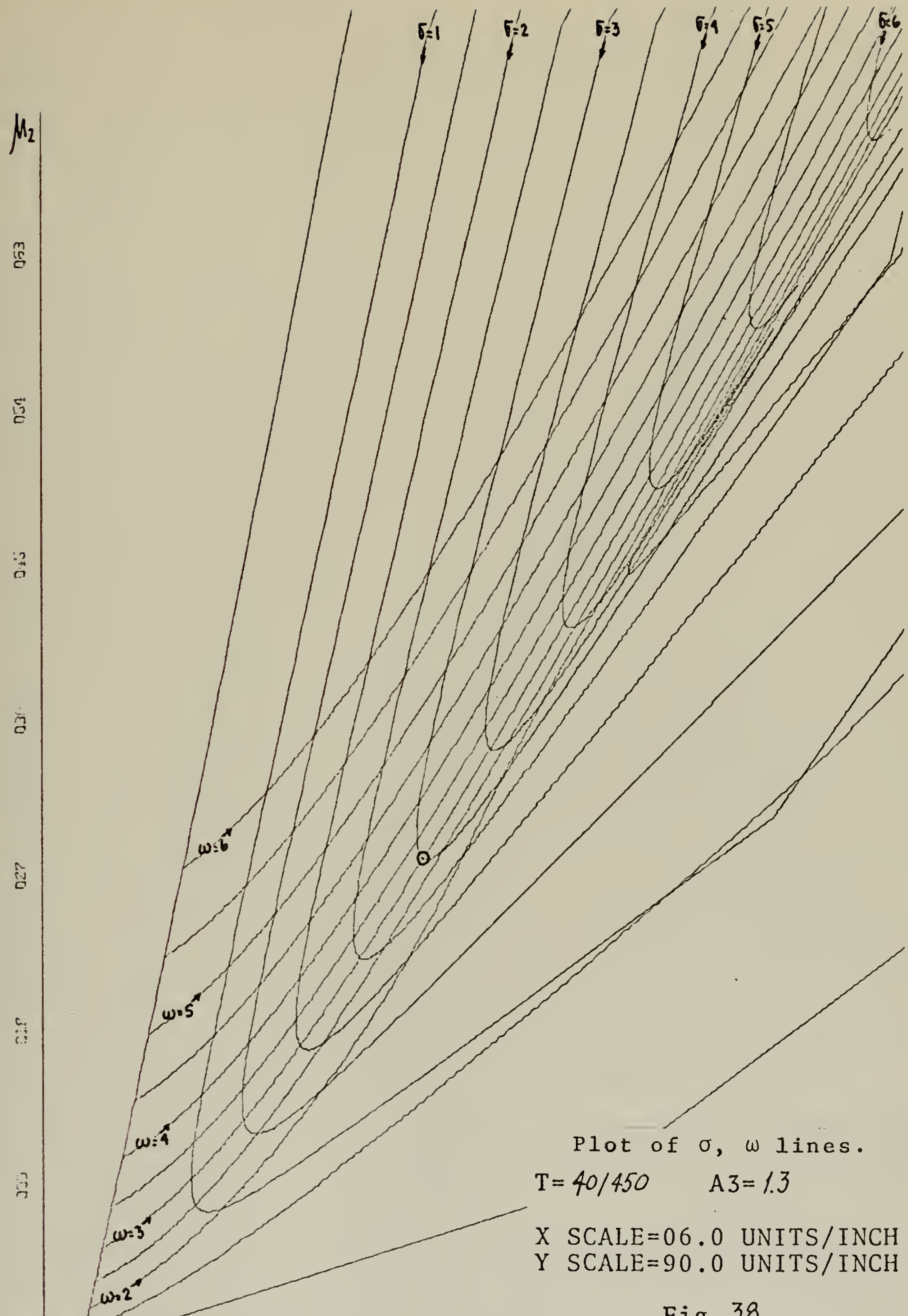


Plot of δ lines.

T= 40/450 A3= 0.5

X SCALE=06.0 UNITS/INCH
Y SCALE=90.0 UNITS/INCH

Fig. 37

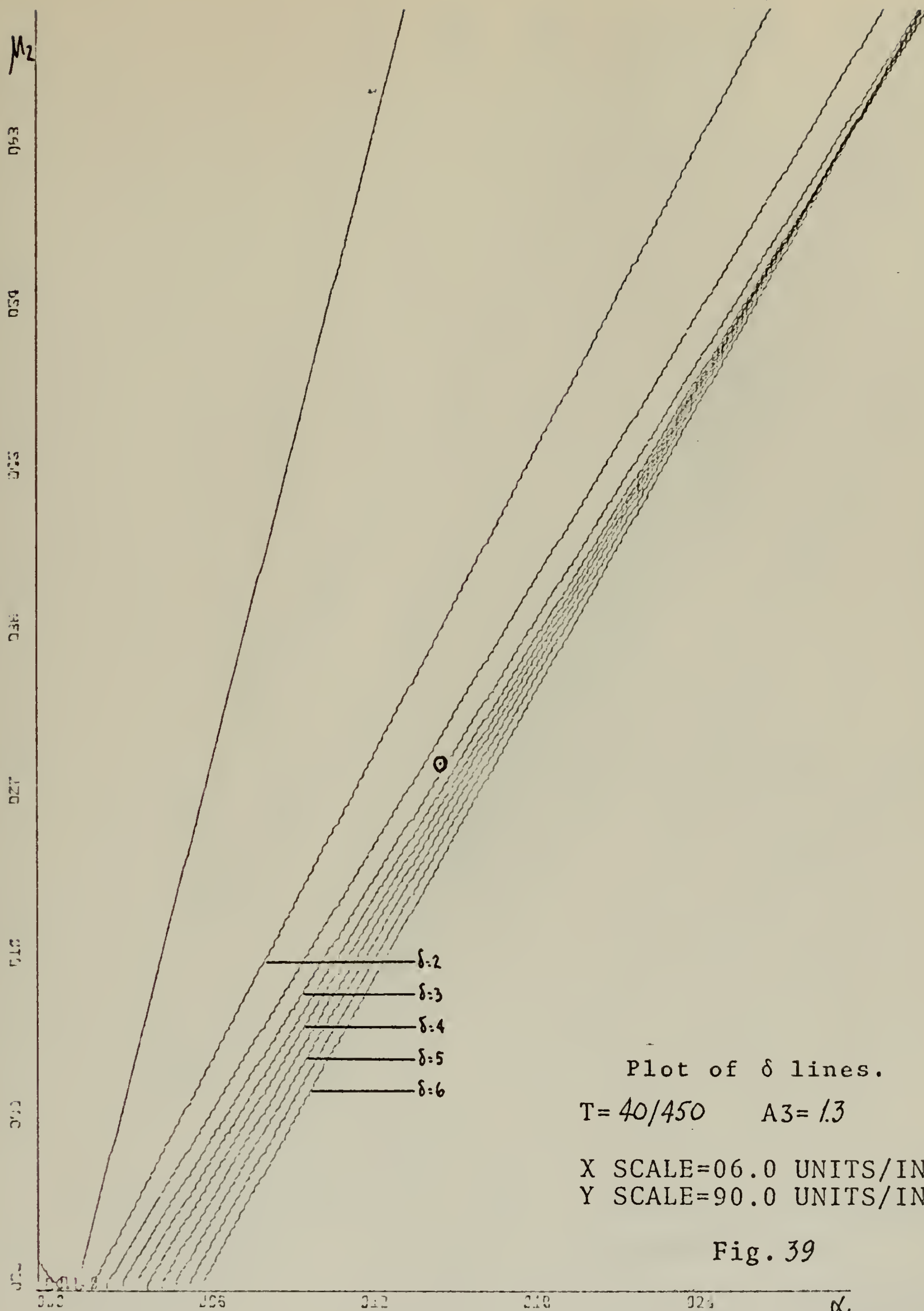


Plot of σ, ω lines.

T= 40/450   A3= 1.3

X SCALE=06.0 UNITS/INCH
Y SCALE=90.0 UNITS/INCH

Fig. 38



Plot of δ lines.

T= 40/450 A3= 1.3

X SCALE=06.0 UNITS/INCH
Y SCALE=90.0 UNITS/INCH

Fig. 39

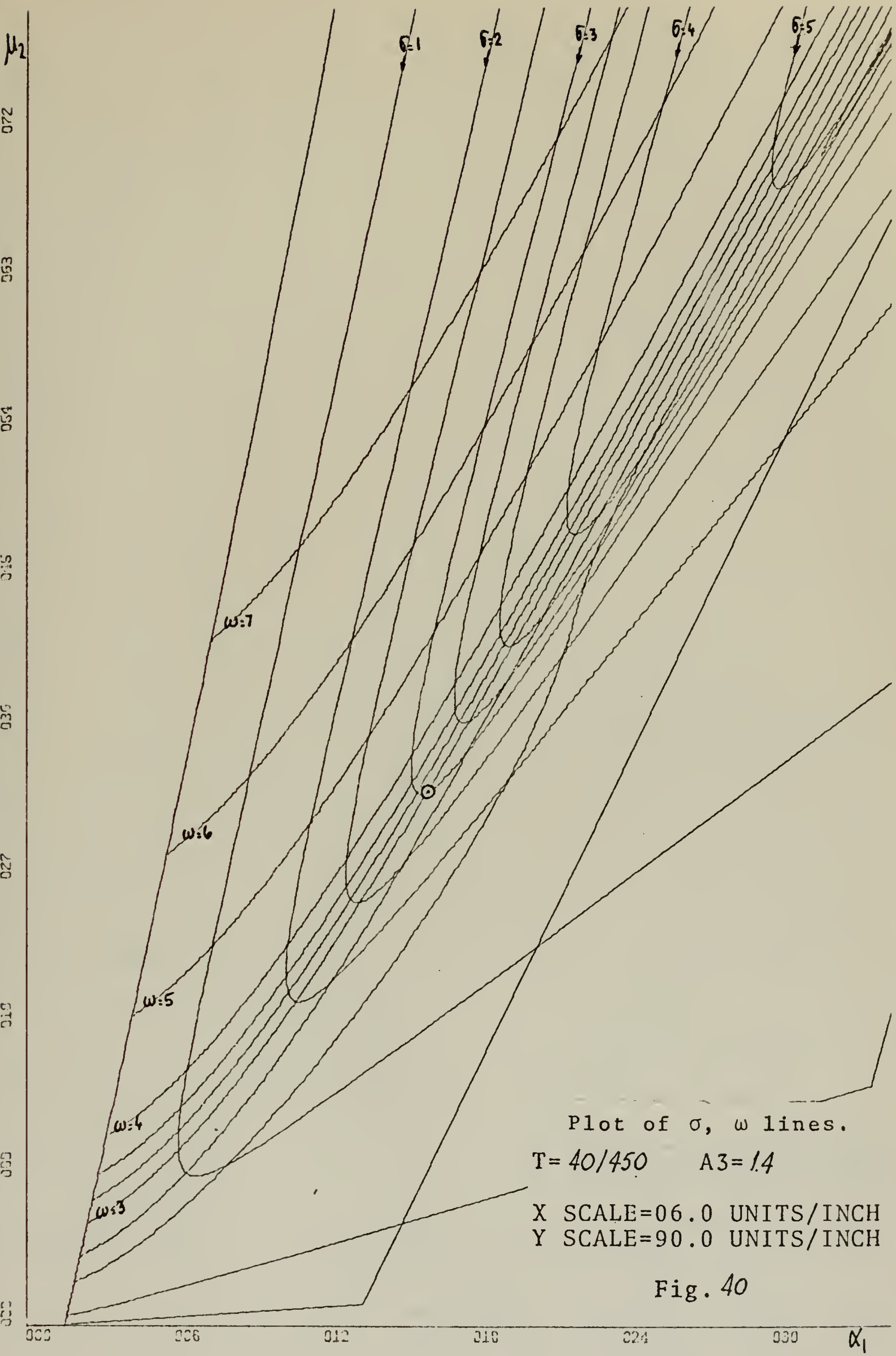
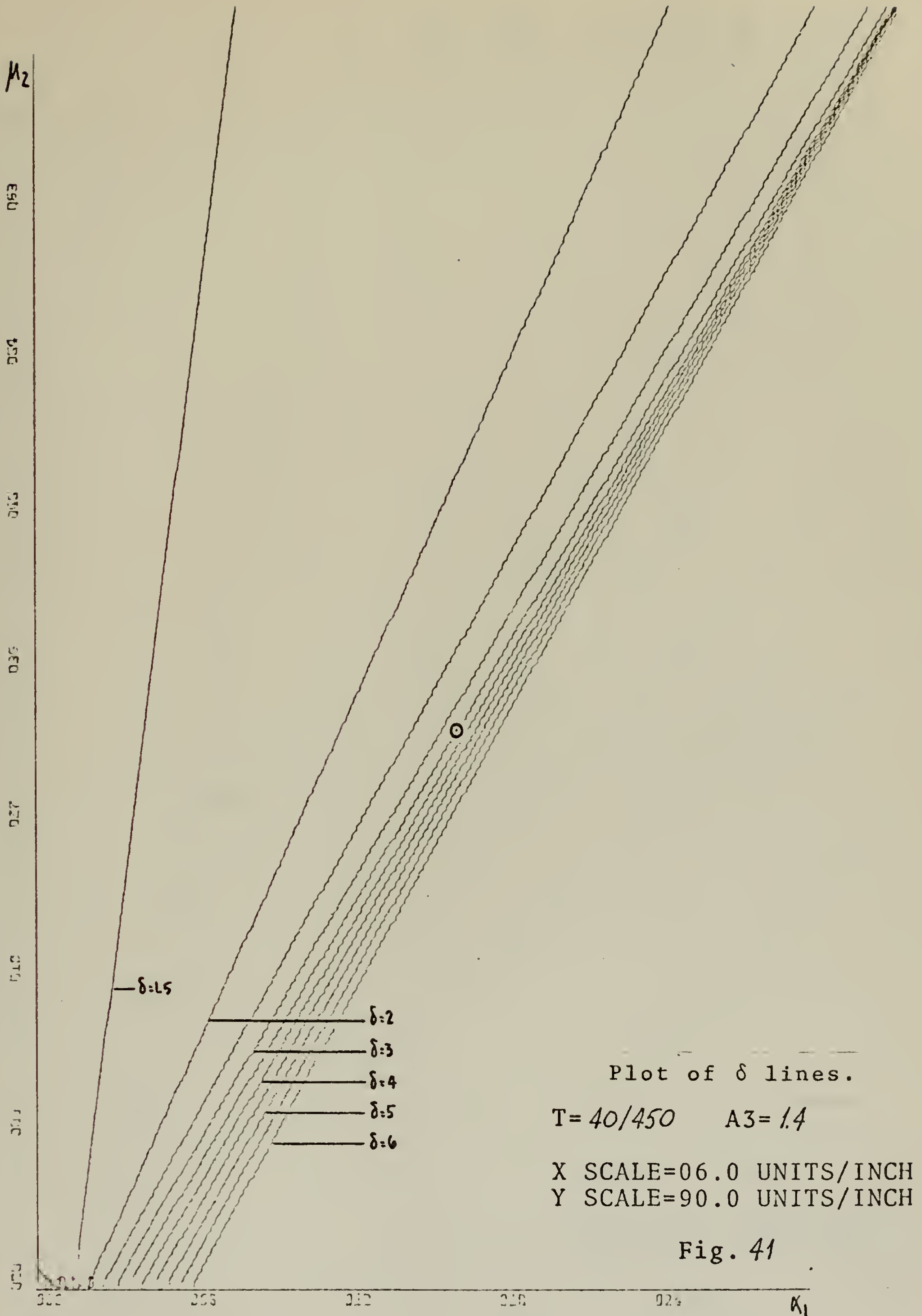

μ_2
072
063
054
027
σ=1
σ=2
σ=3
σ=4
σ=5
ω=7
ω=6
ω=5
ω=4
ω=3
000
006
012
018
024
030
α_1
Plot of σ, ω lines.
T= 40/450 A3= 1.4
X SCALE=06.0 UNITS/INCH
Y SCALE=90.0 UNITS/INCH

Fig. 40



Plot of δ lines.

T= 40/450 A3= 1.4

X SCALE=06.0 UNITS/INCH
Y SCALE=90.0 UNITS/INCH

Fig. 41

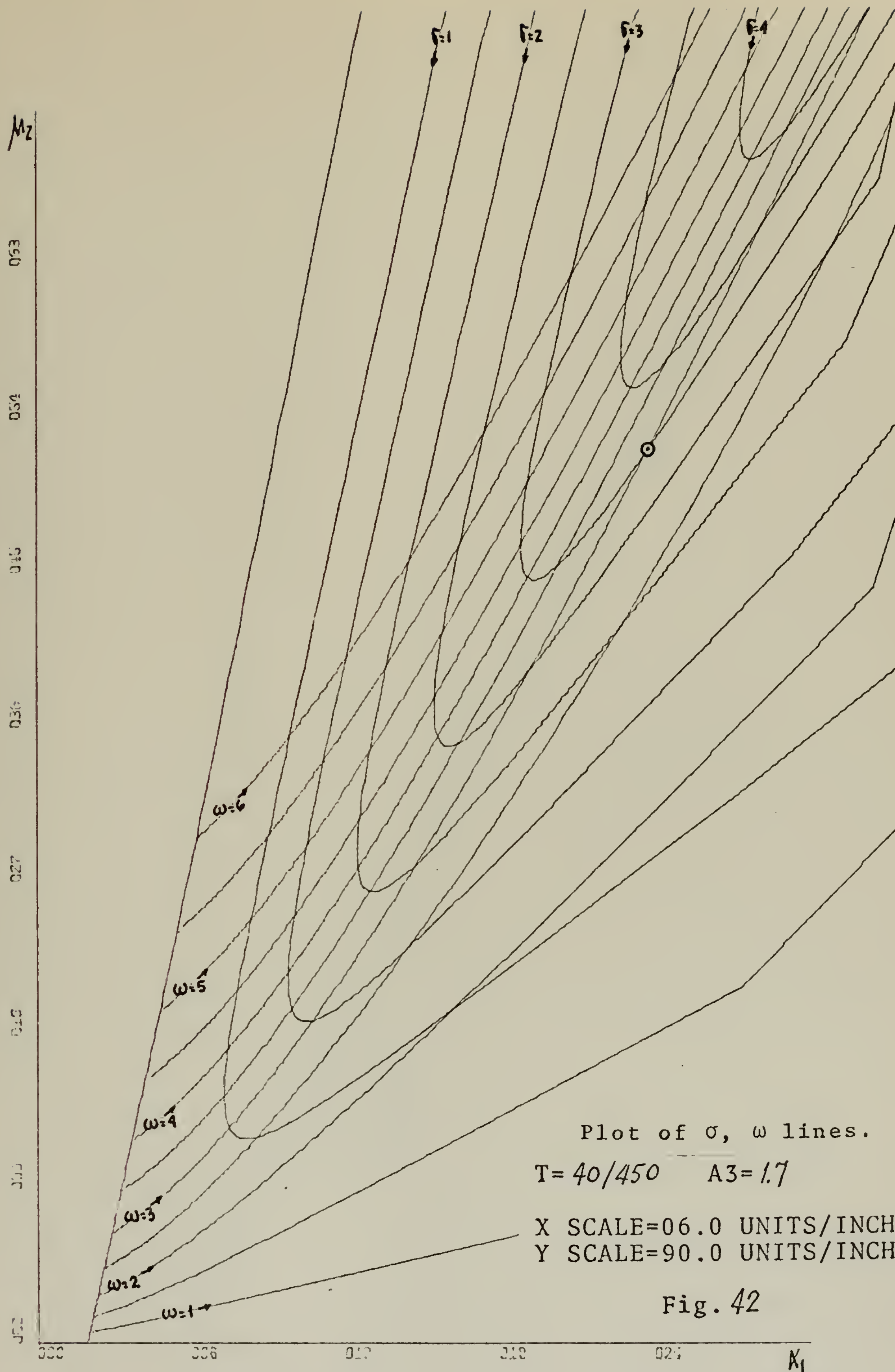

σ=1
σ=2
σ=3
σ=4
ω=1
ω=2
ω=3
ω=4
ω=5
ω=6
Plot of σ, ω lines.
T= 40/450 A3= 1.7
X SCALE=06.0 UNITS/INCH
Y SCALE=90.0 UNITS/INCH

Fig. 42

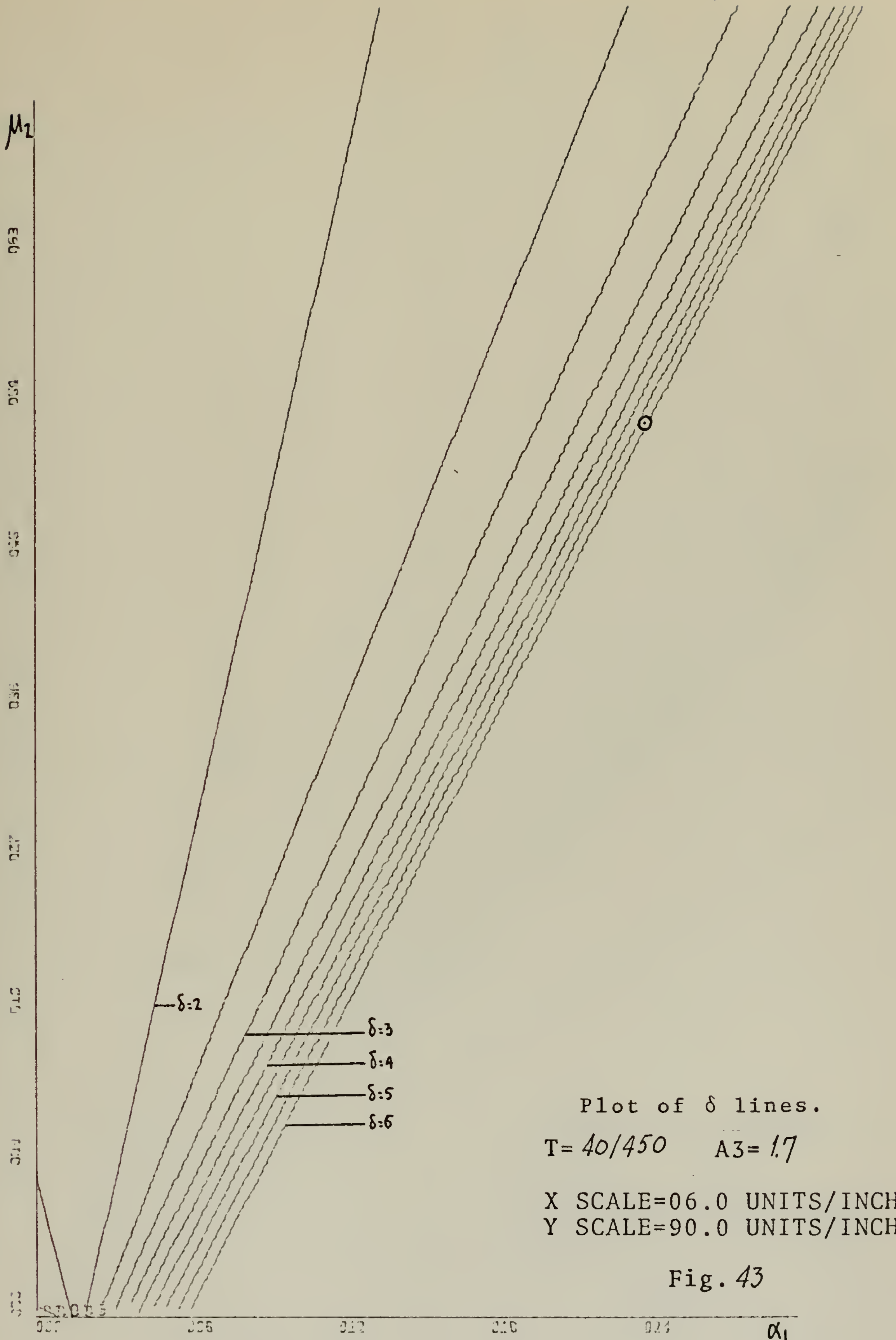
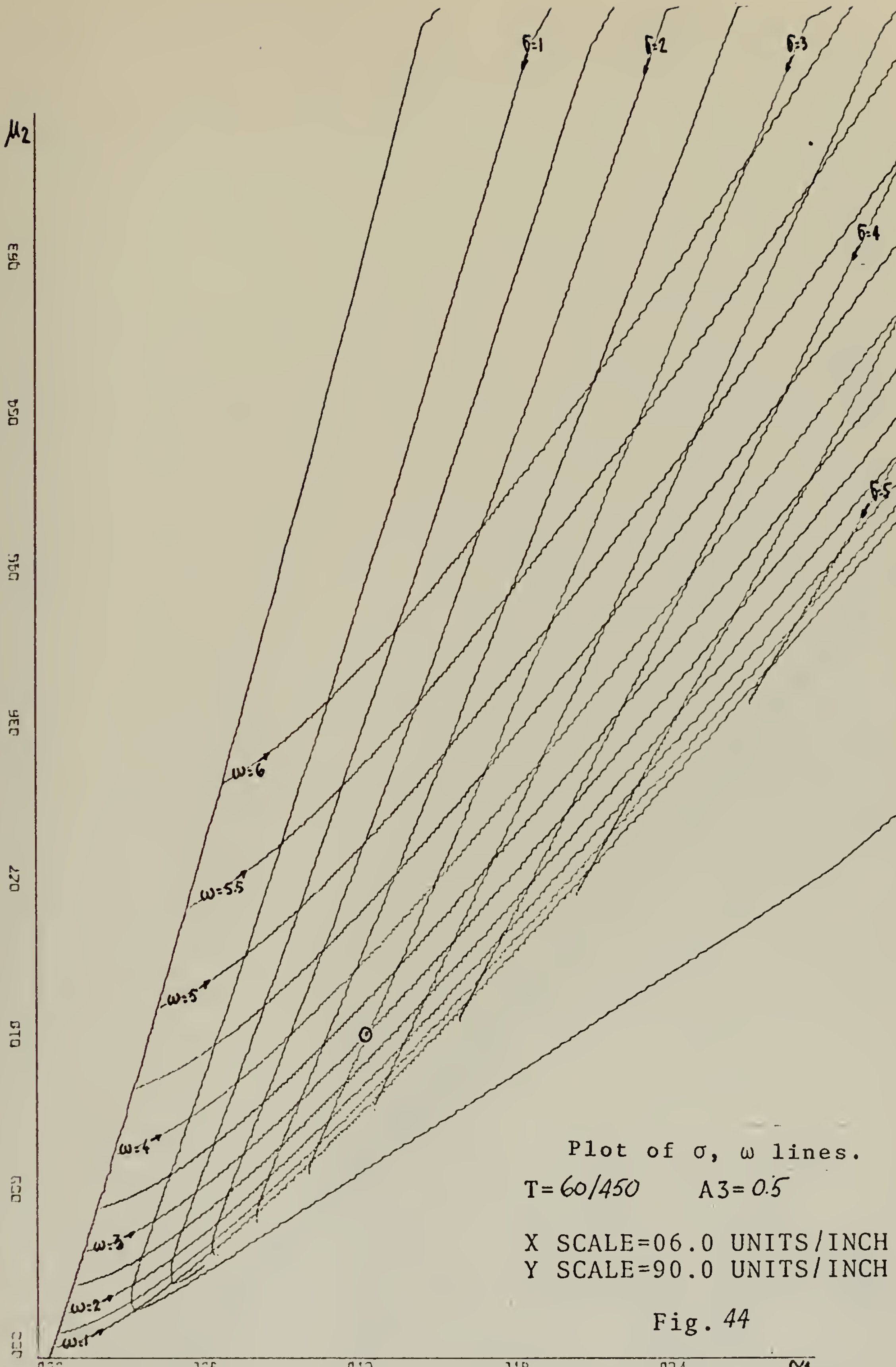

μ2
δ=2
δ=3
δ=4
δ=5
δ=6
Plot of δ lines.
T= 40/450 A3= 1.7
X SCALE=06.0 UNITS/INCH
Y SCALE=90.0 UNITS/INCH
α1

Fig. 43

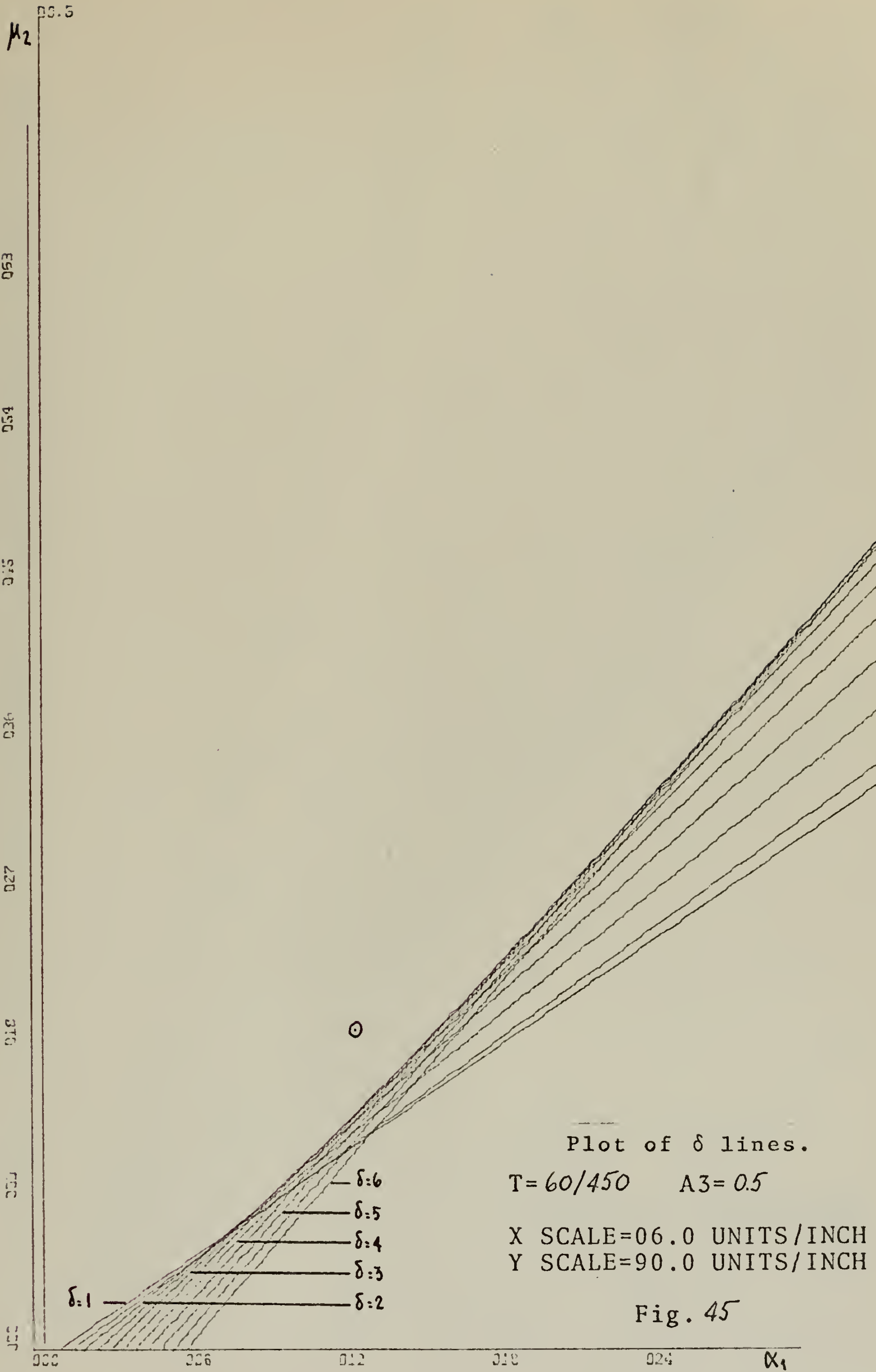


Plot of σ, ω lines.

T=60/450 A3=0.5

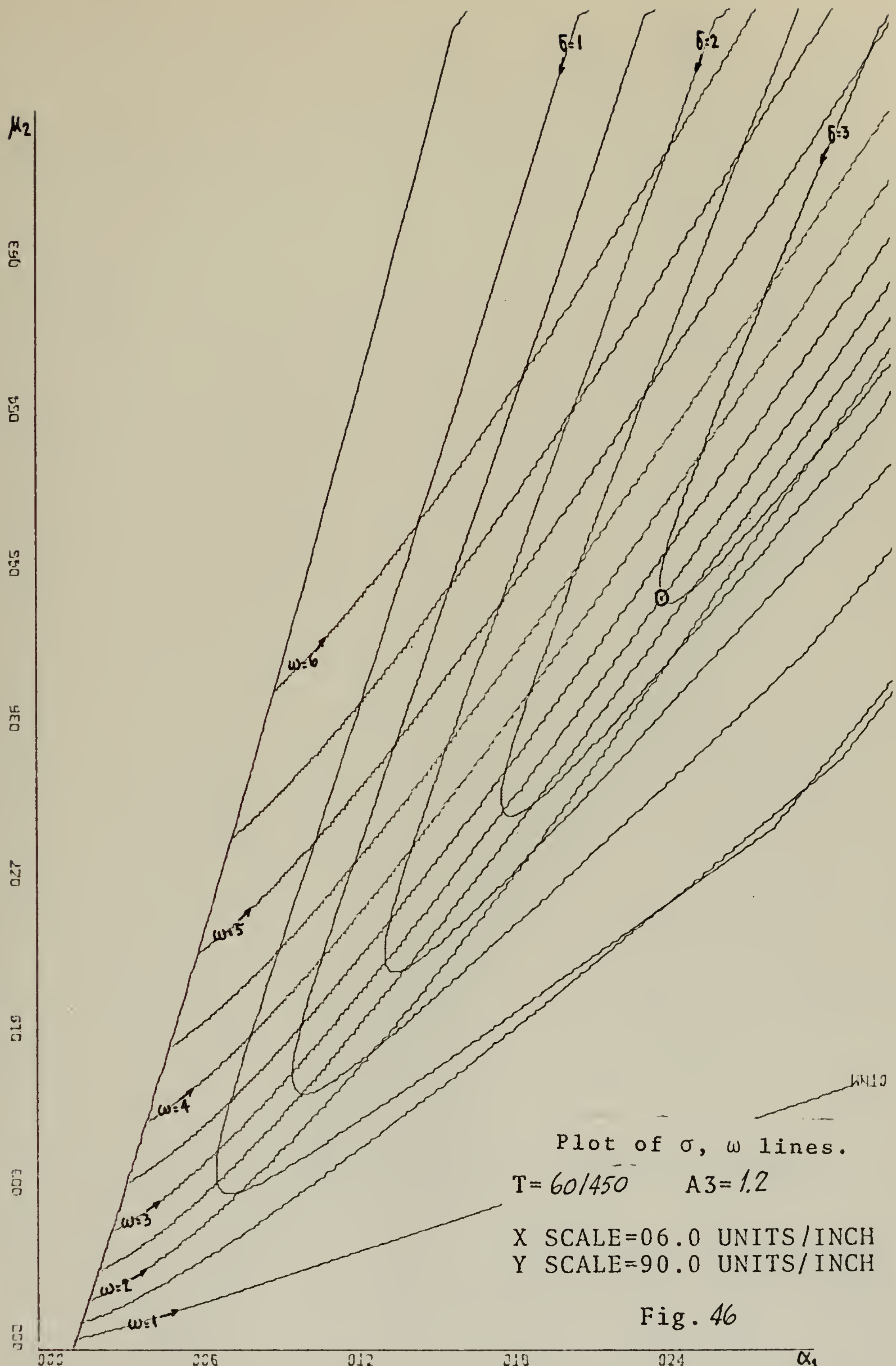
X SCALE=06.0 UNITS/INCH
Y SCALE=90.0 UNITS/INCH

Fig. 44



μ_2
α_1
δ:1
δ:2
δ:3
δ:4
δ:5
δ:6
Plot of δ lines.
T= 60/450 A3= 0.5
X SCALE=06.0 UNITS/INCH
Y SCALE=90.0 UNITS/INCH

Fig. 45

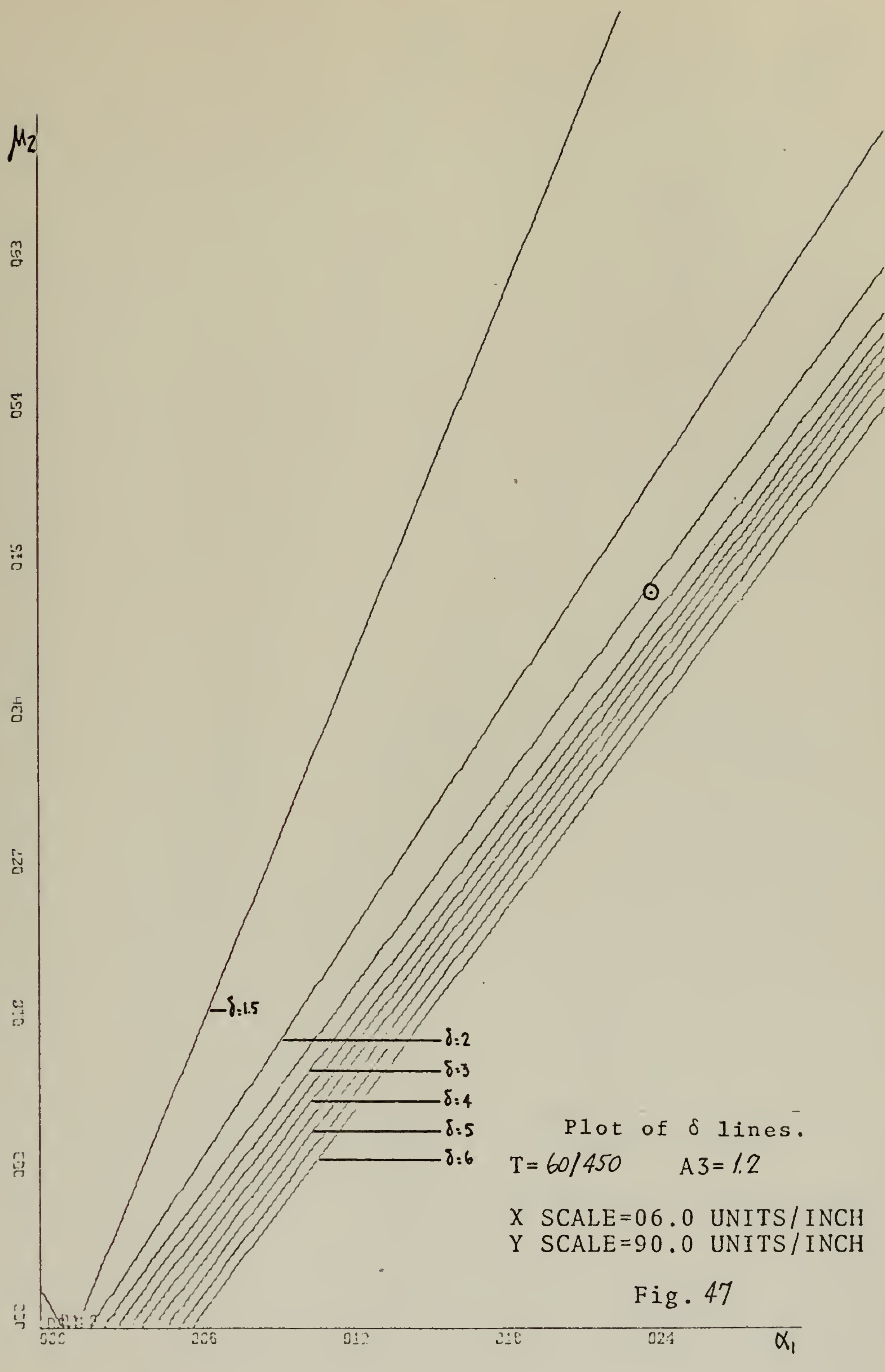


Plot of σ, ω lines.

T= 60/450 A3= 1.2

X SCALE=06.0 UNITS/INCH
Y SCALE=90.0 UNITS/INCH

Fig. 46

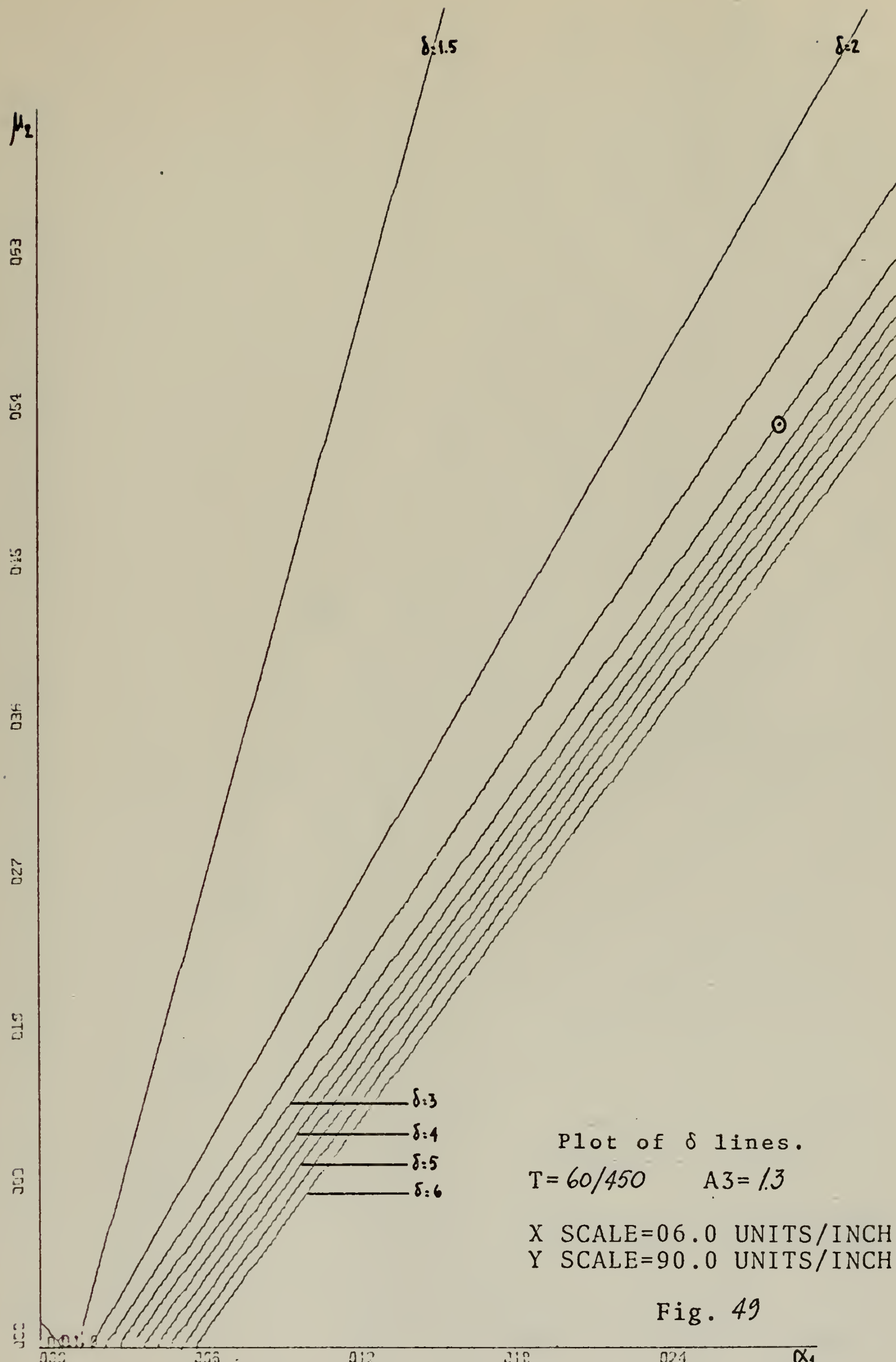


Plot of δ lines.

T= 60/450 A3= 1.2

X SCALE=06.0 UNITS/INCH
Y SCALE=90.0 UNITS/INCH

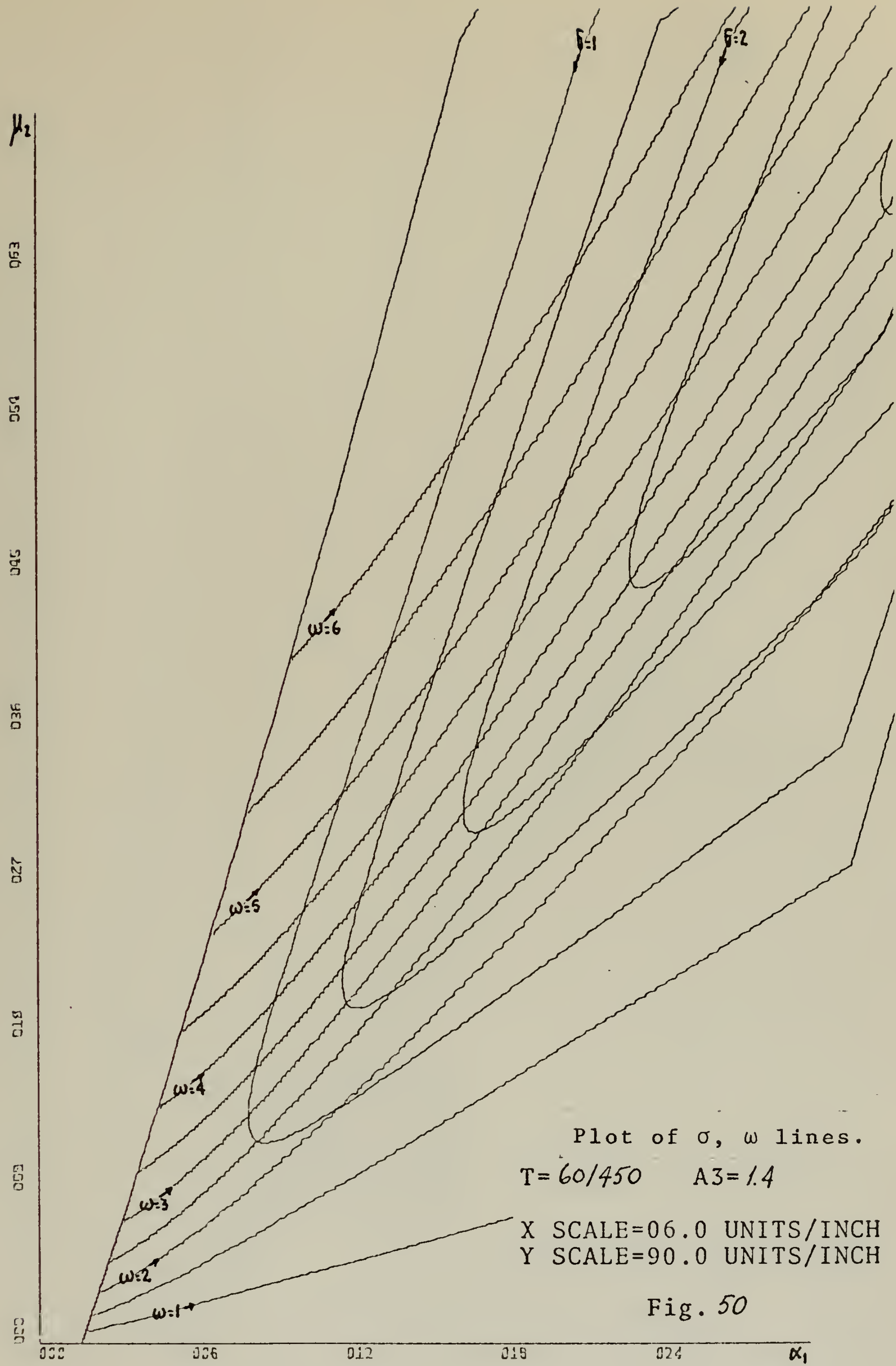
Fig. 47



Plot of δ lines.
T= 60/450   A3= /.3

X SCALE=06.0 UNITS/INCH
Y SCALE=90.0 UNITS/INCH

Fig. 49

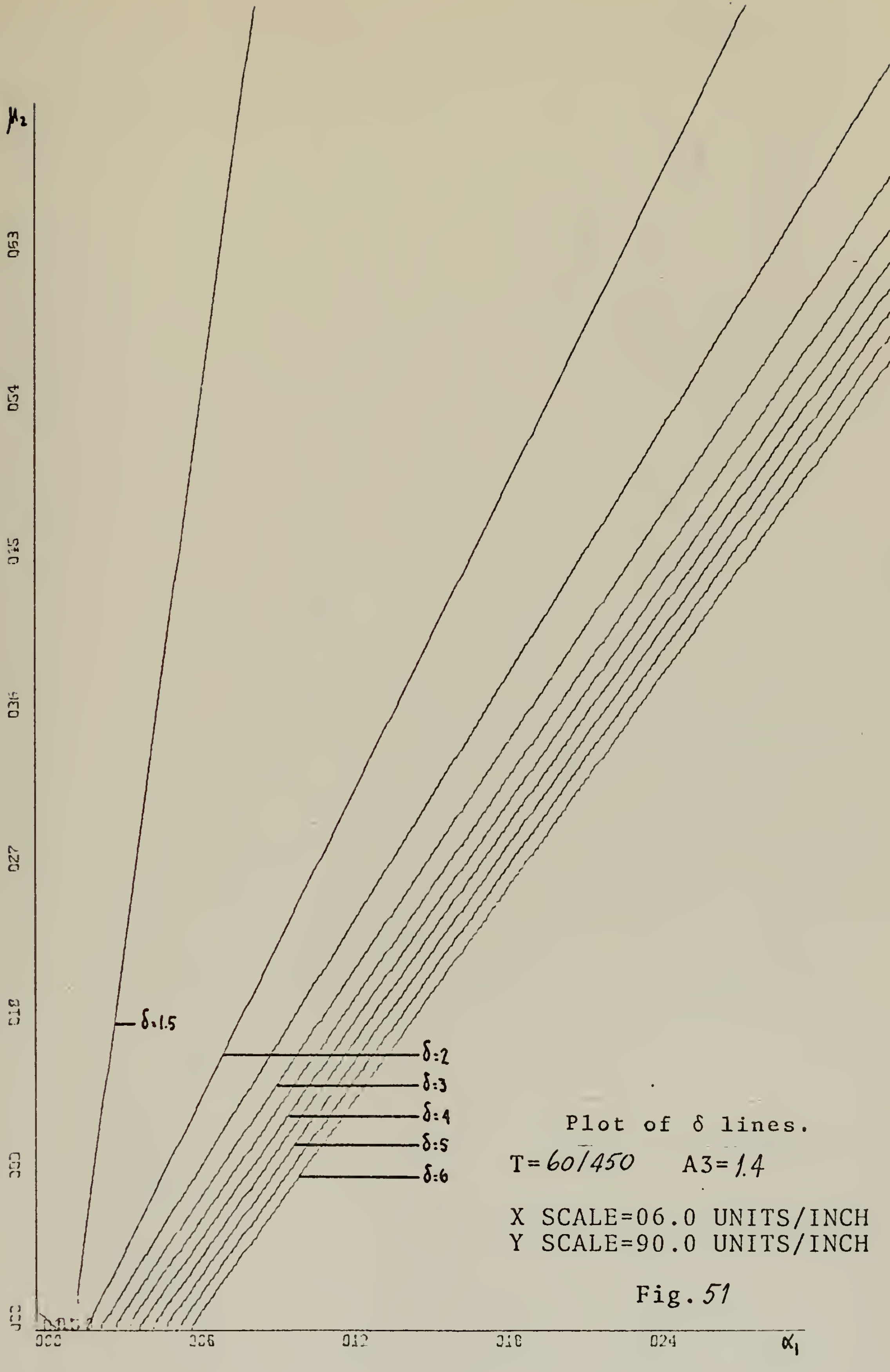


Plot of σ, ω lines.

T= 60/450   A3= 1.4

X SCALE=06.0 UNITS/INCH
Y SCALE=90.0 UNITS/INCH

Fig. 50

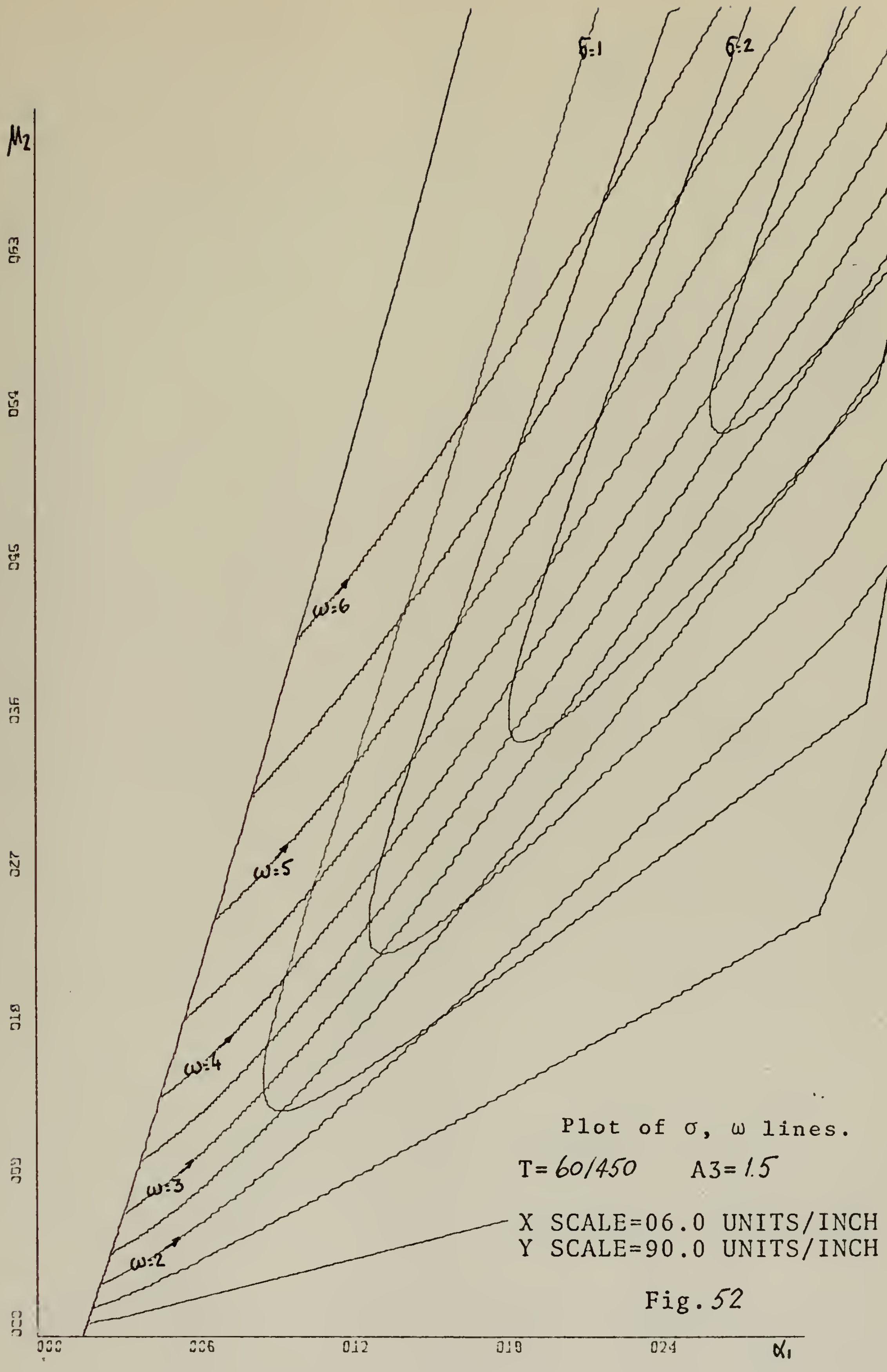


Plot of δ lines.

T=60/450 A3=1.4

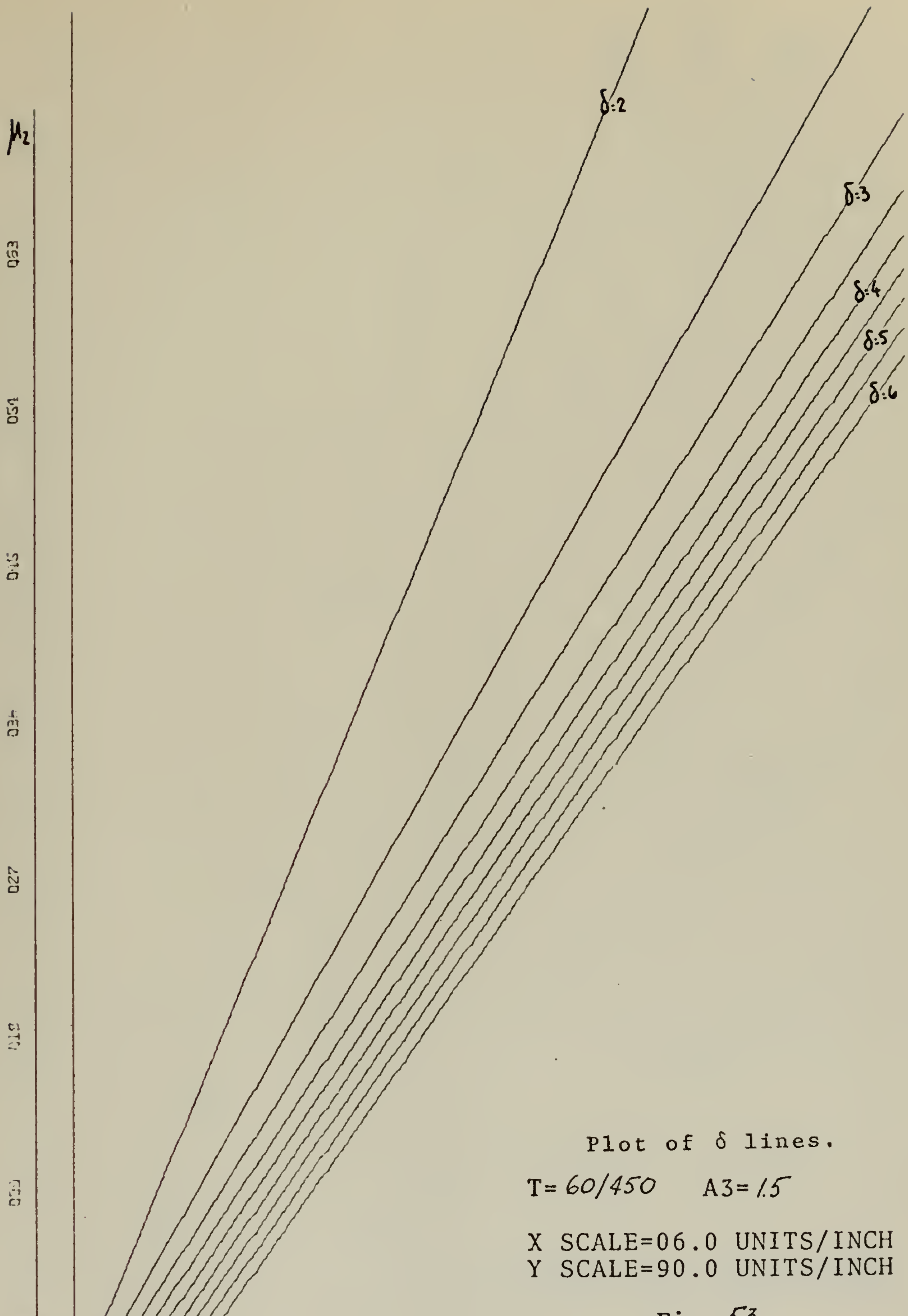
X SCALE=06.0 UNITS/INCH
Y SCALE=90.0 UNITS/INCH

Fig. 51



μ_2
063
054
045
036
027
018
009
000
000
006
012
018
024
α_1
σ=1
σ=2
ω=6
ω=5
ω=4
ω=3
ω=2
Plot of σ, ω lines.
T=60/450 A3=1.5
X SCALE=06.0 UNITS/INCH
Y SCALE=90.0 UNITS/INCH

Fig. 52

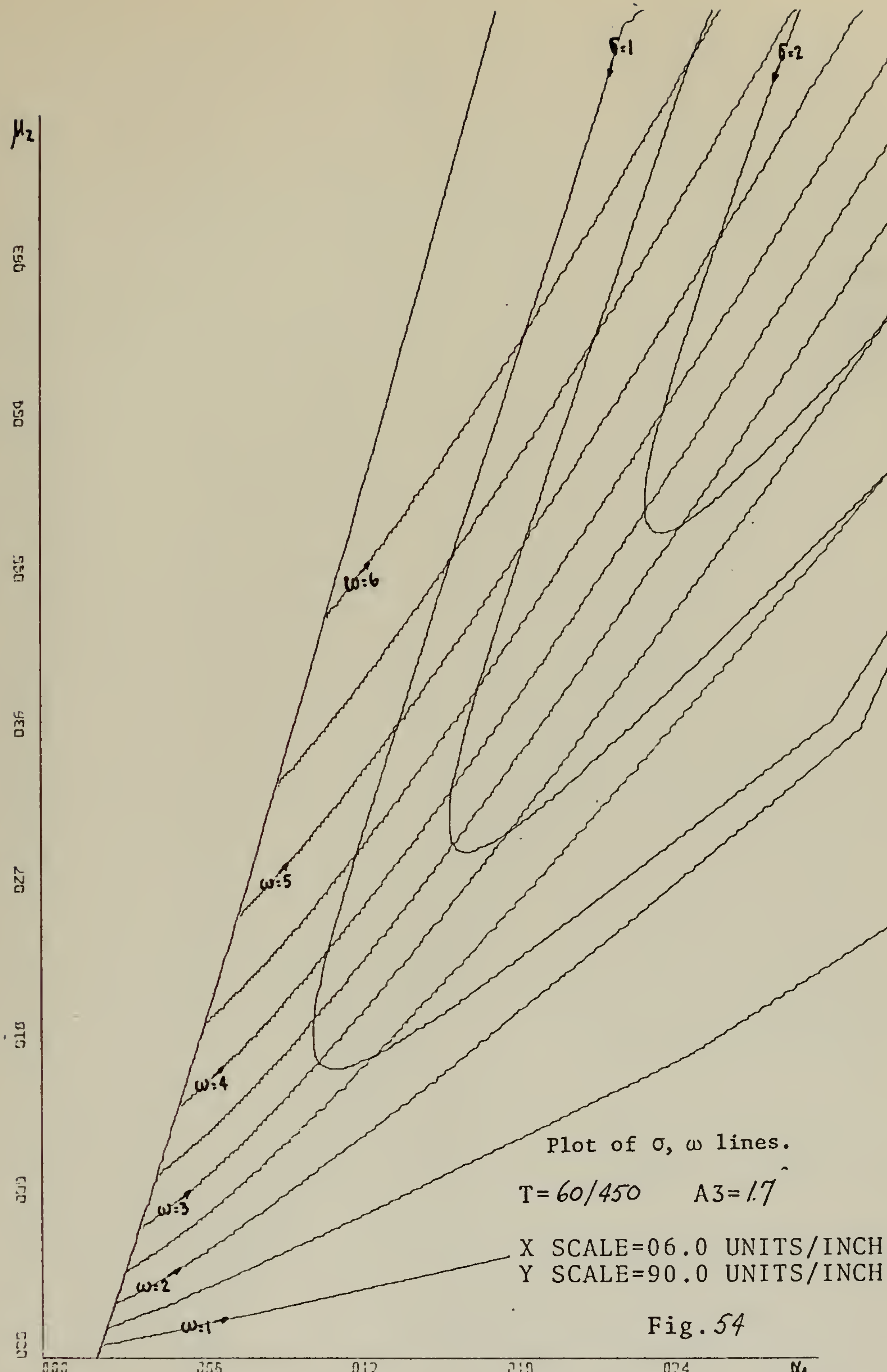


Plot of δ lines.

T= 60/450 A3= 1.5

X SCALE=06.0 UNITS/INCH
Y SCALE=90.0 UNITS/INCH

Fig. 53



μ_2
α_1
027
036
000
006
012
018
024
ω=1
ω=2
ω=3
ω=4
ω=5
ω=6
σ=1
σ=2
Plot of σ, ω lines.
T=60/450 A3=1.7
X SCALE=06.0 UNITS/INCH
Y SCALE=90.0 UNITS/INCH

Fig. 54

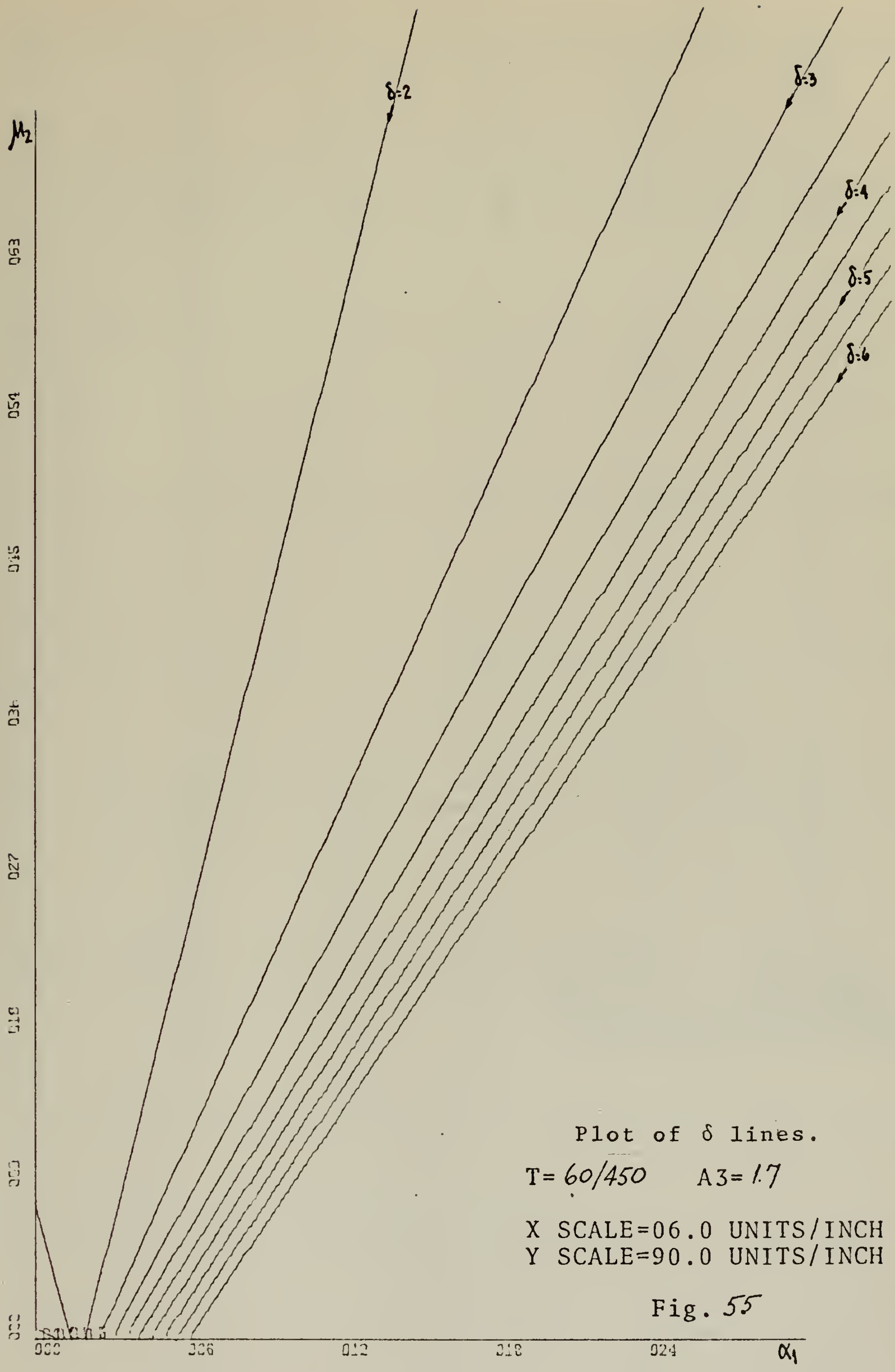

μ_2
063
054
045
036
027
018
δ=2
δ=3
δ=4
δ=5
δ=6
000
006
012
018
024
α_1
Plot of δ lines.
T= 60/450 A3= 1.7
X SCALE=06.0 UNITS/INCH
Y SCALE=90.0 UNITS/INCH

Fig. 55

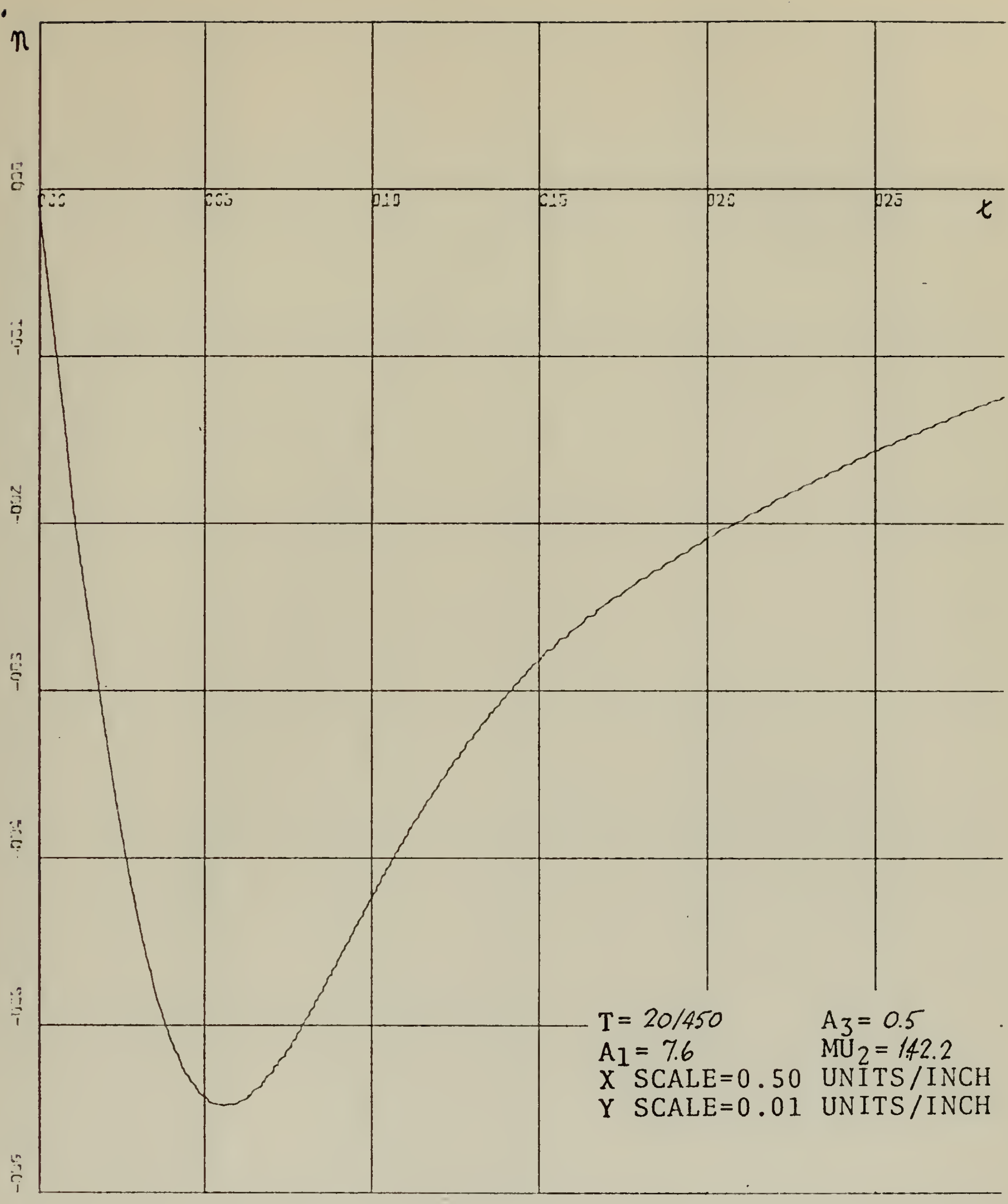


Figure 56. Speed transient of Diesel.

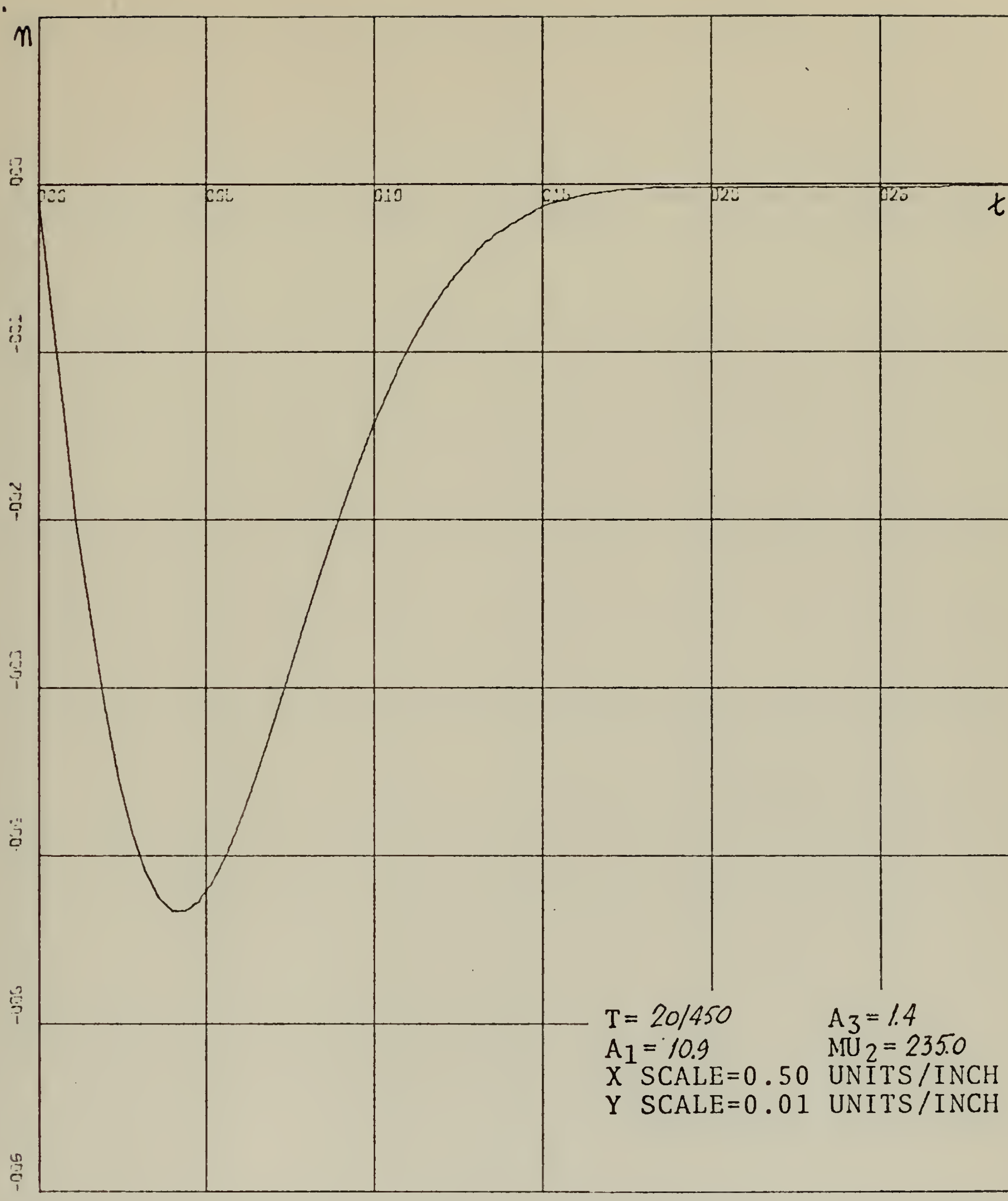


Figure 57. Speed transient of Diesel.

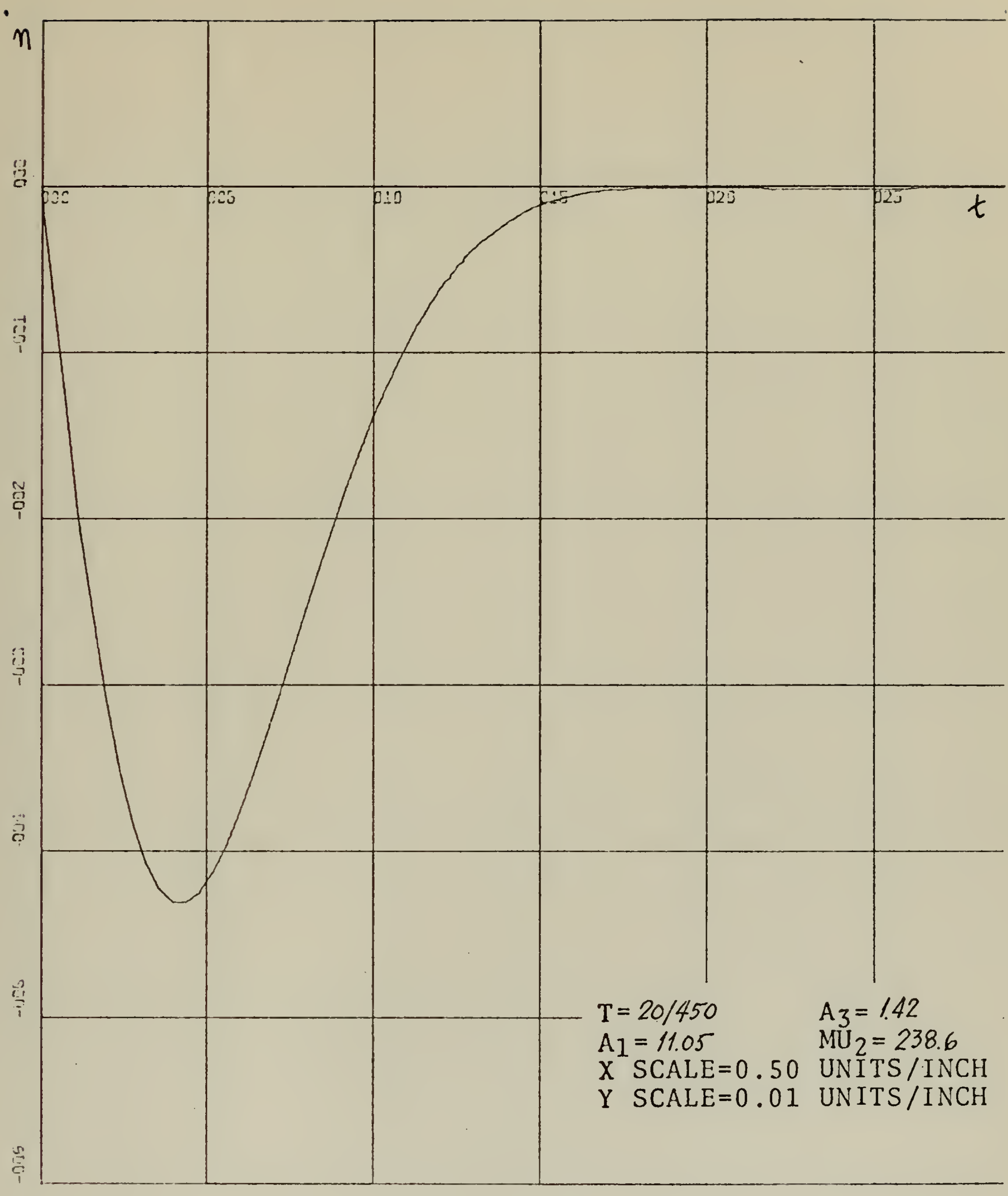


Figure 58. Speed transient of Diesel.

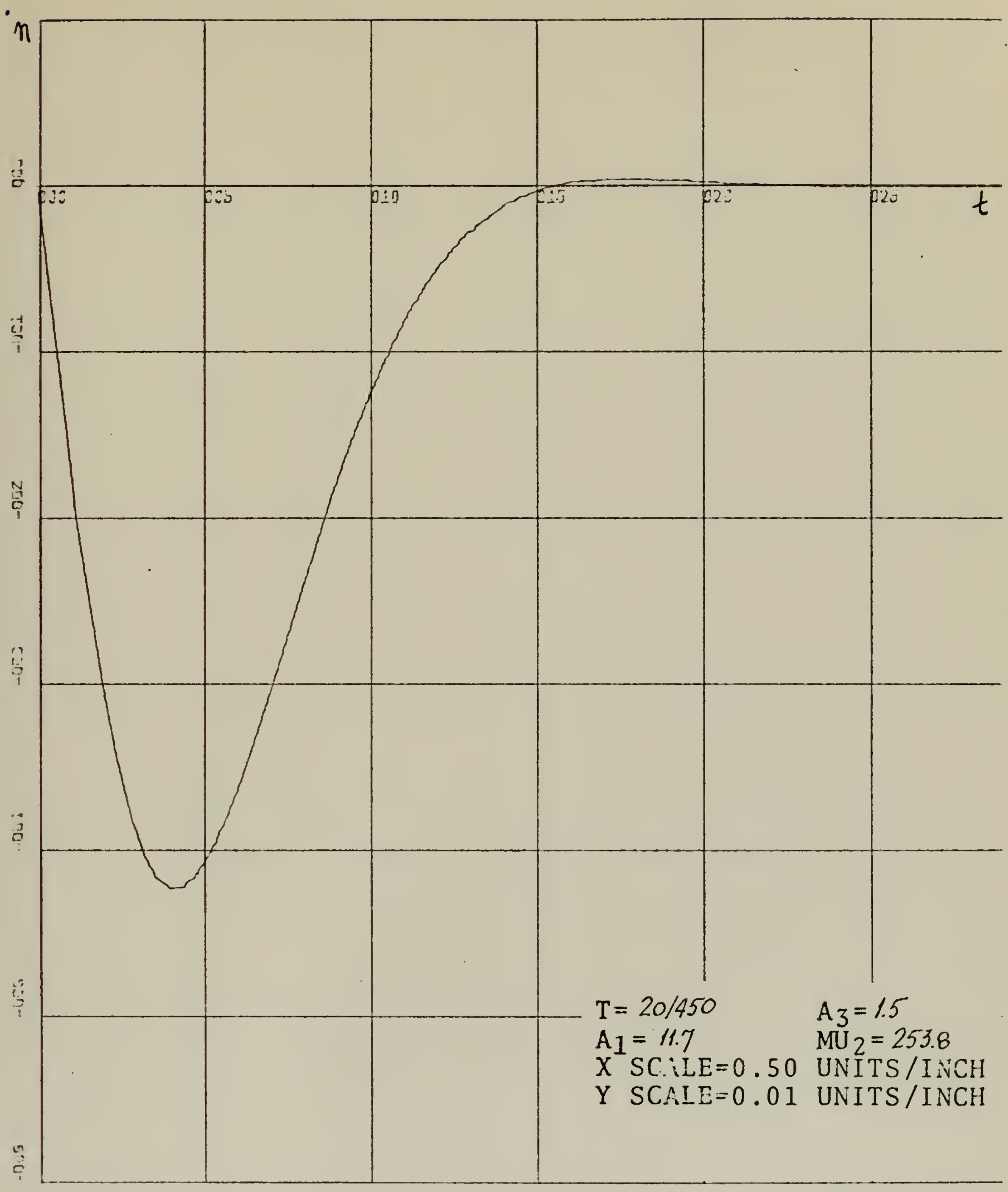


Figure 59. Speed transient of Diesel.

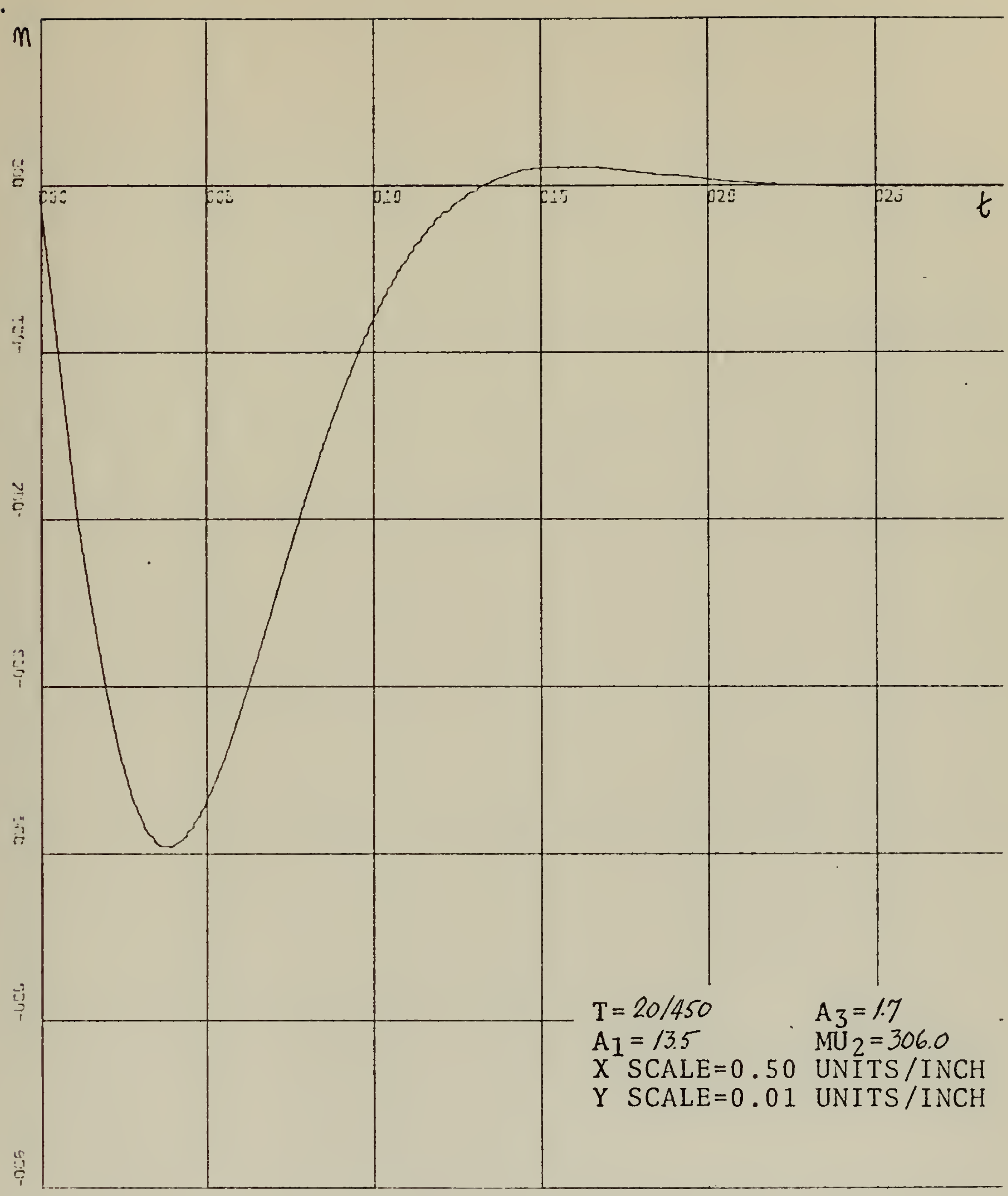


Figure 60. Speed transient of Diesel.

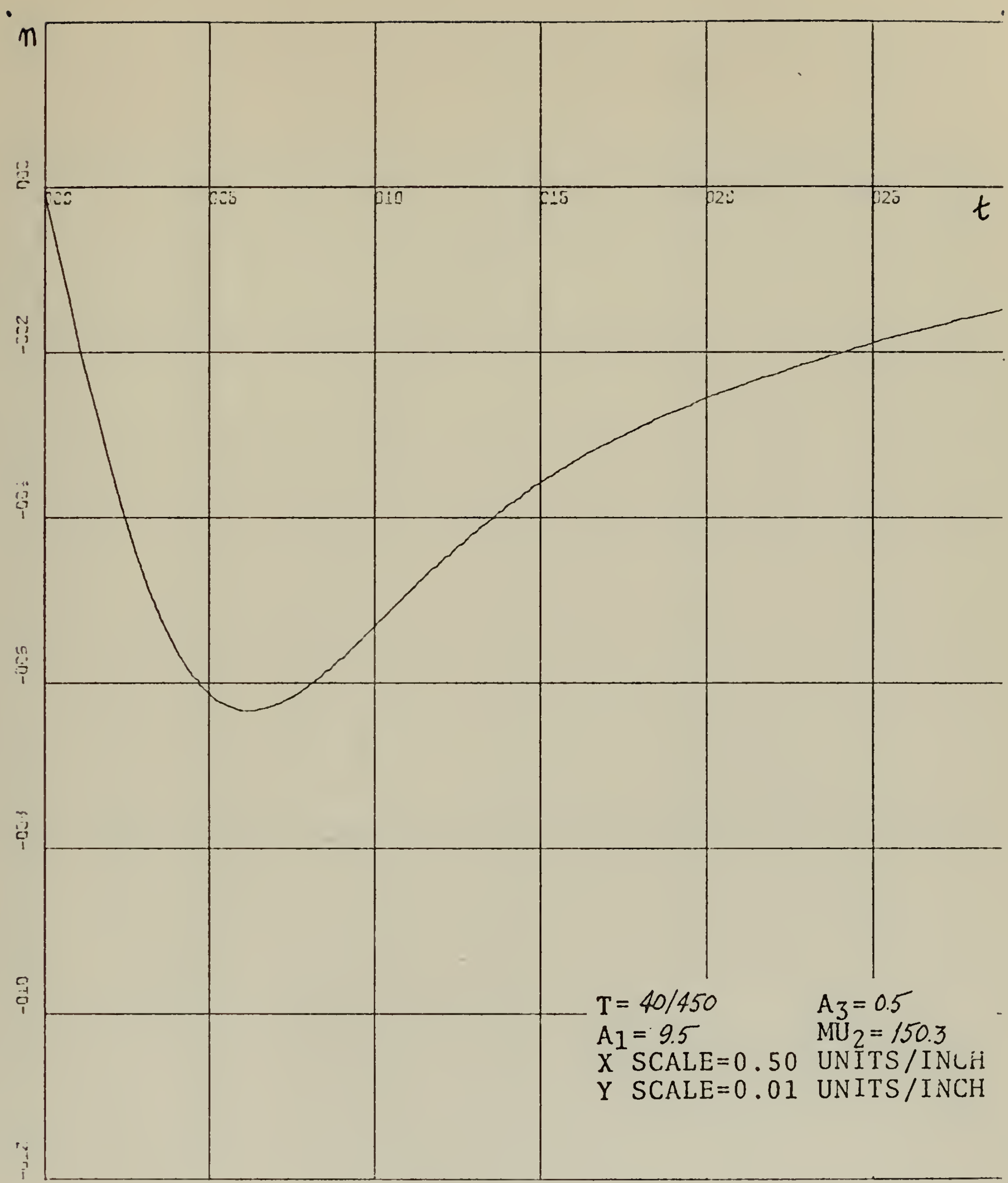


Figure 61. Speed transient of 2-cylinder G.E.

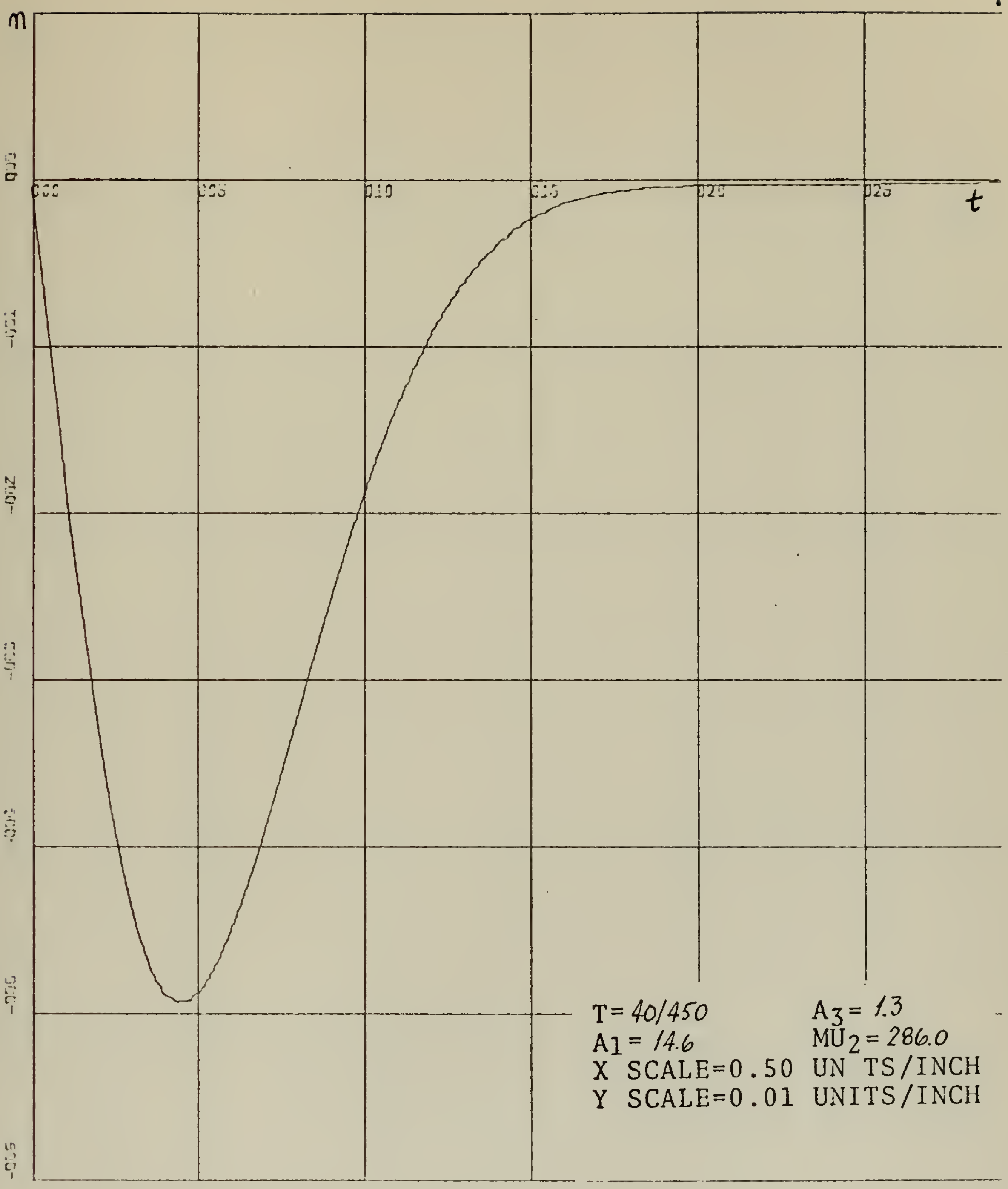


Figure 62. Speed transient of 2-cylinder G.E.

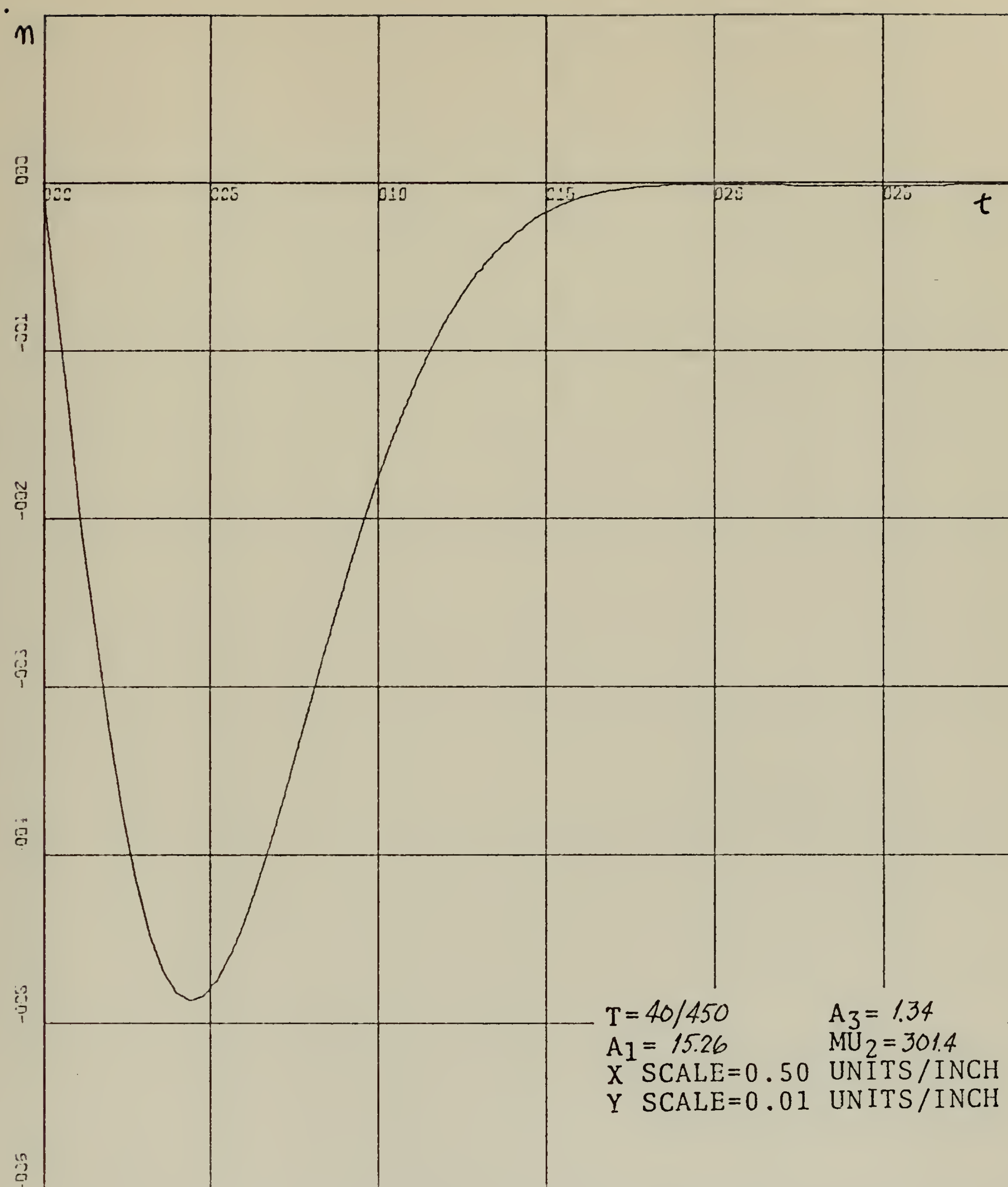


Figure 63. Speed transient of 2-cylinder G.E.

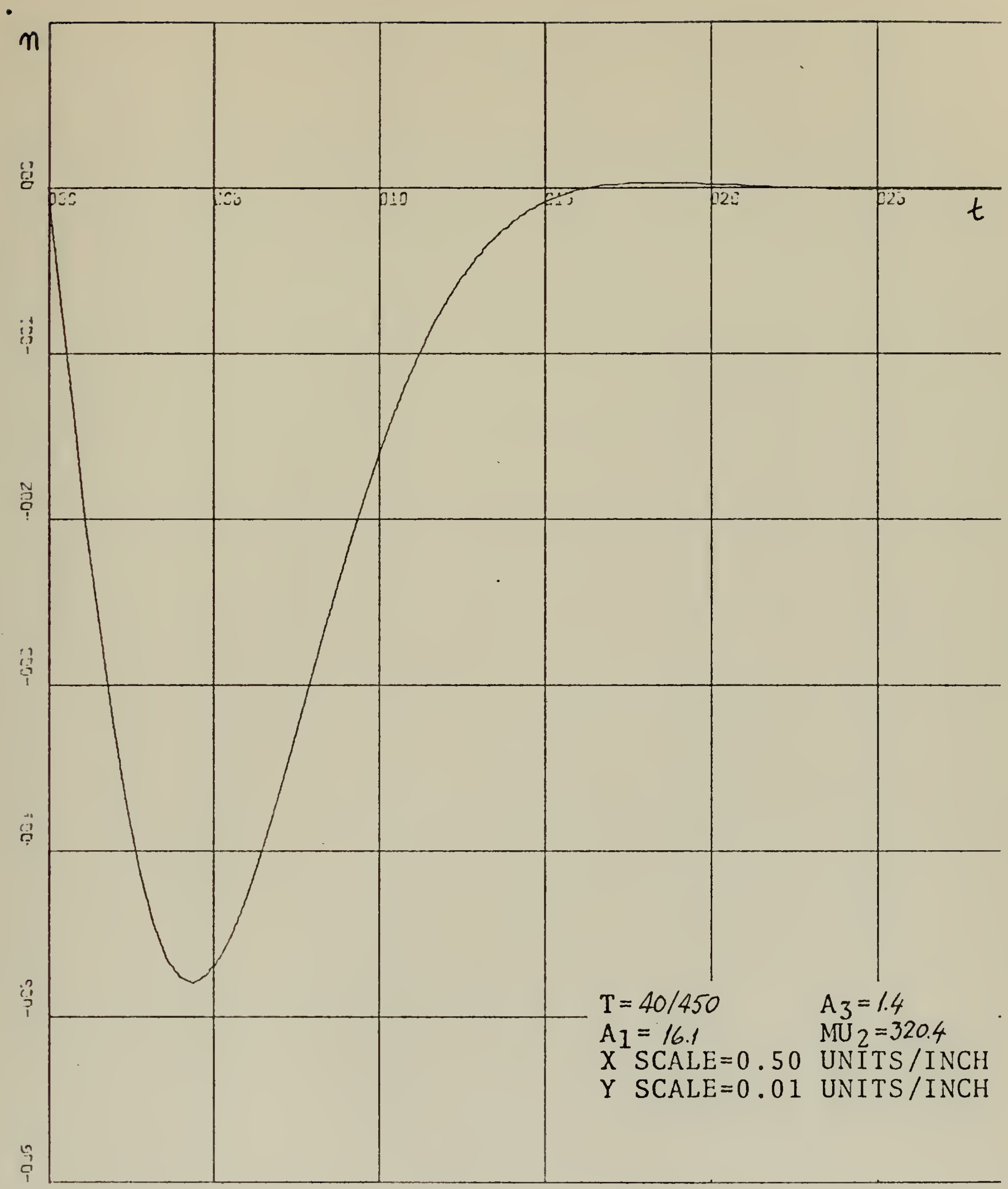


Figure 64. Speed transient of 2-cylinder G.E.

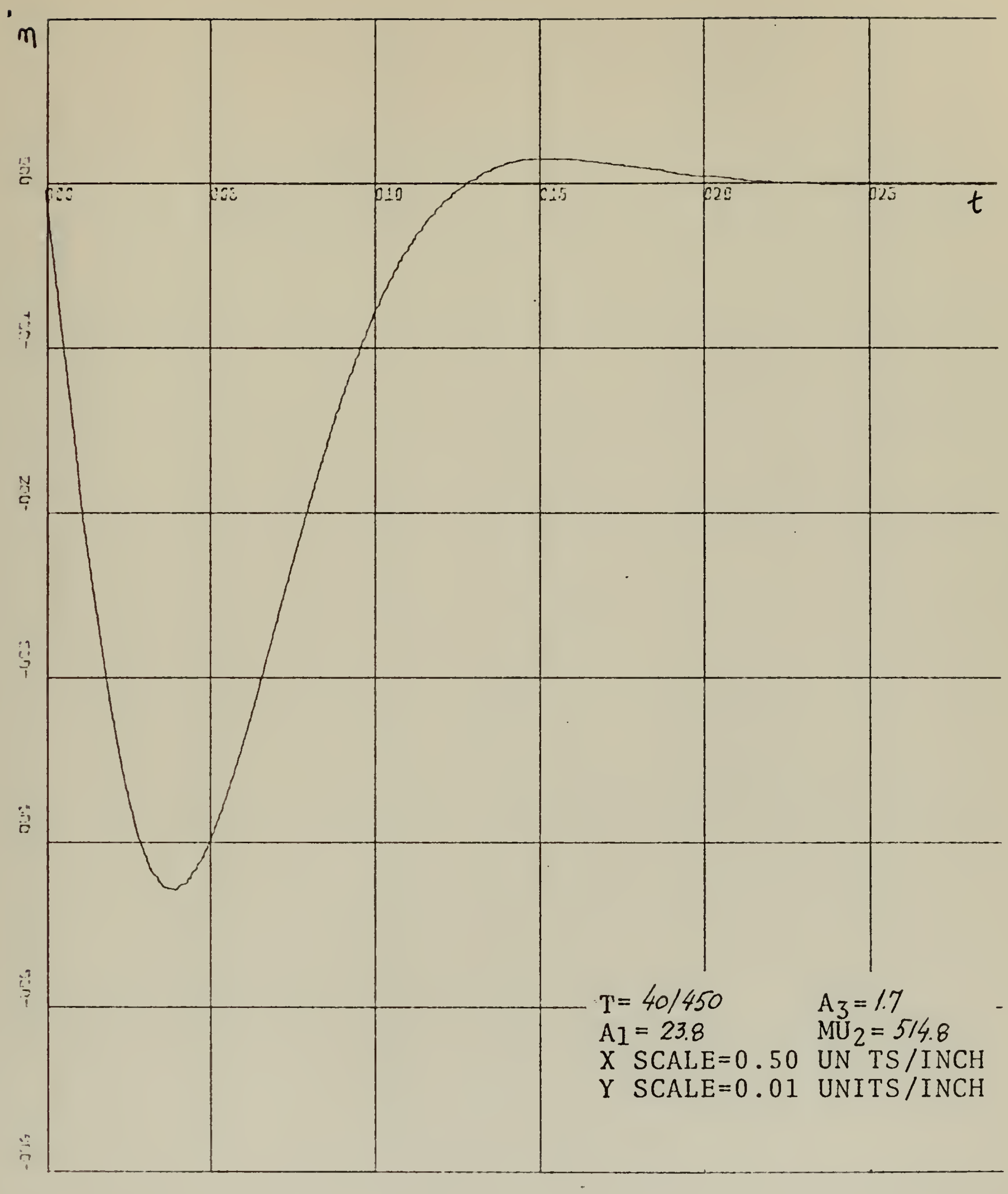


Figure 65. Speed transient of 2-cylinder G.E.

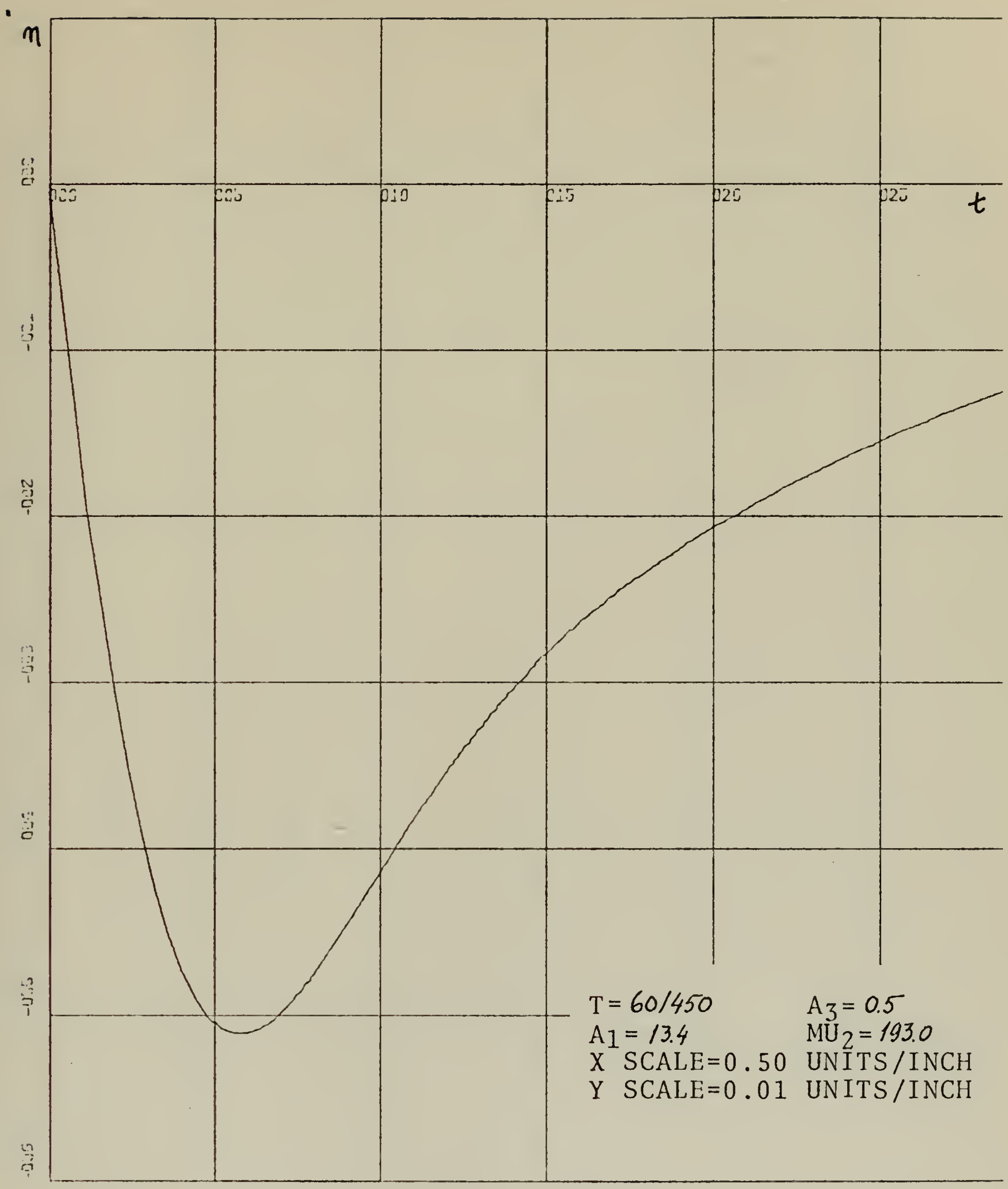


Figure 66. Speed transient of 4-cylinder G.E.

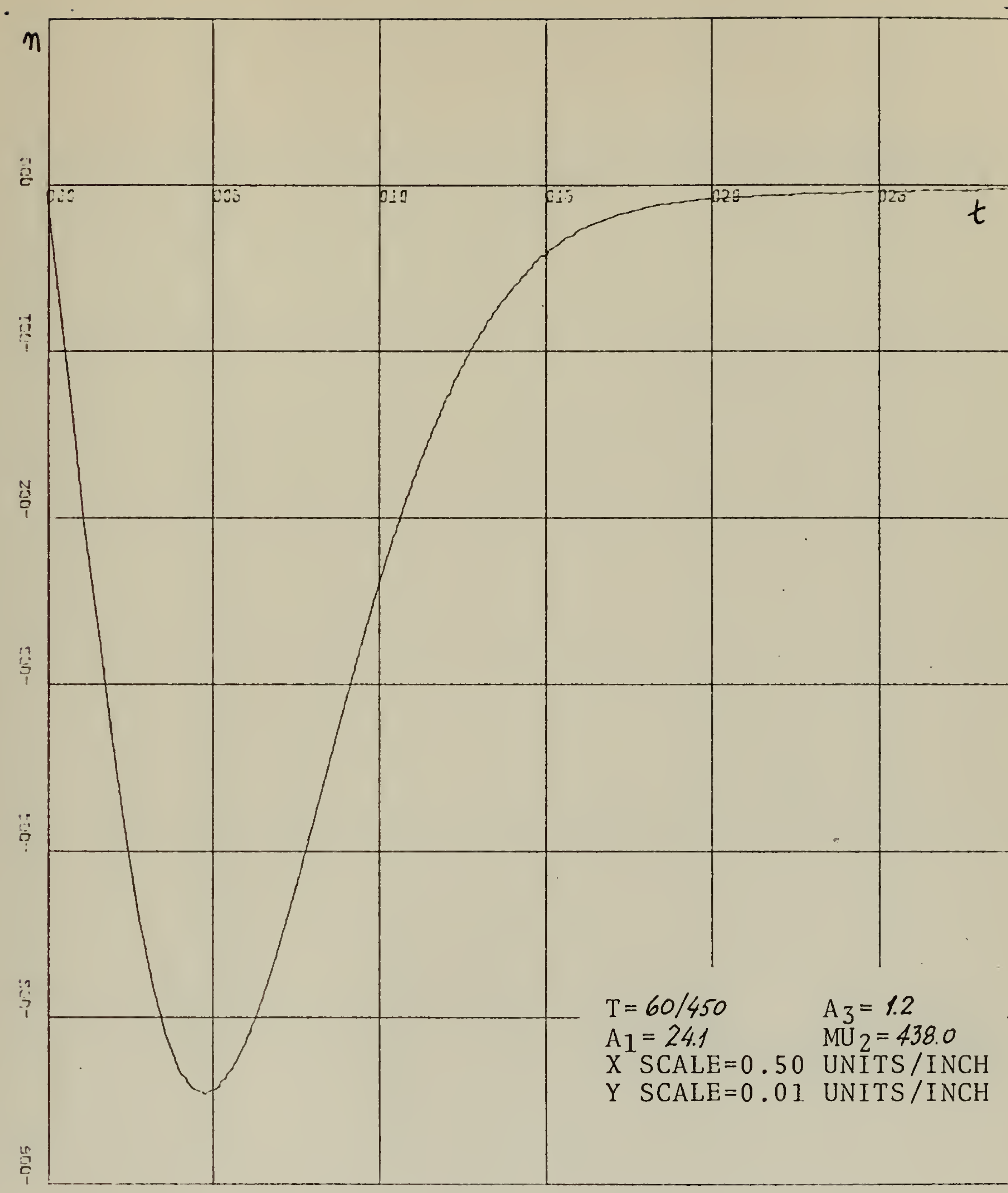


Figure 67. Speed transient of 4-cylinder G.E.

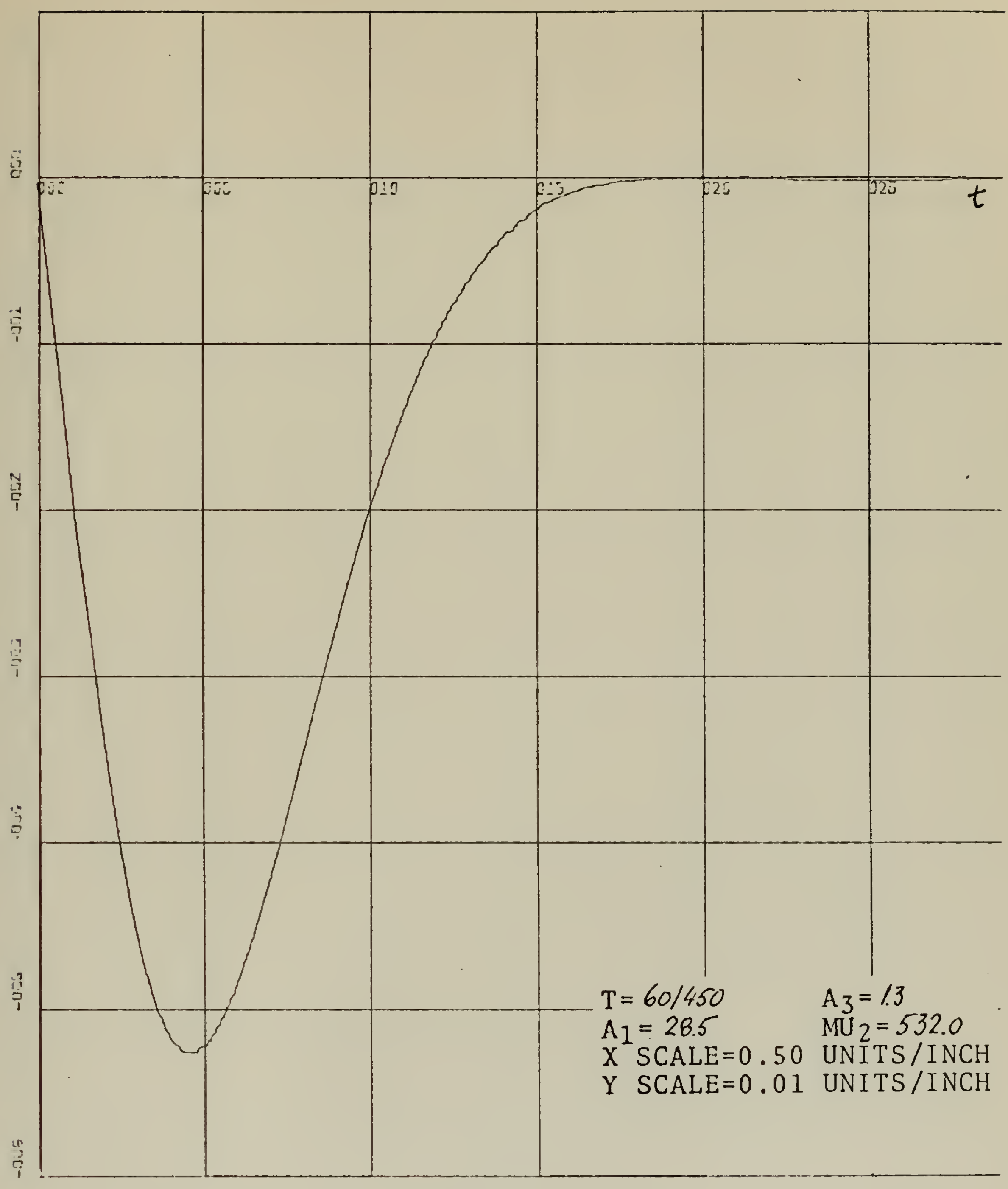


Figure 68. Speed transient of 4-cylinder G.E.

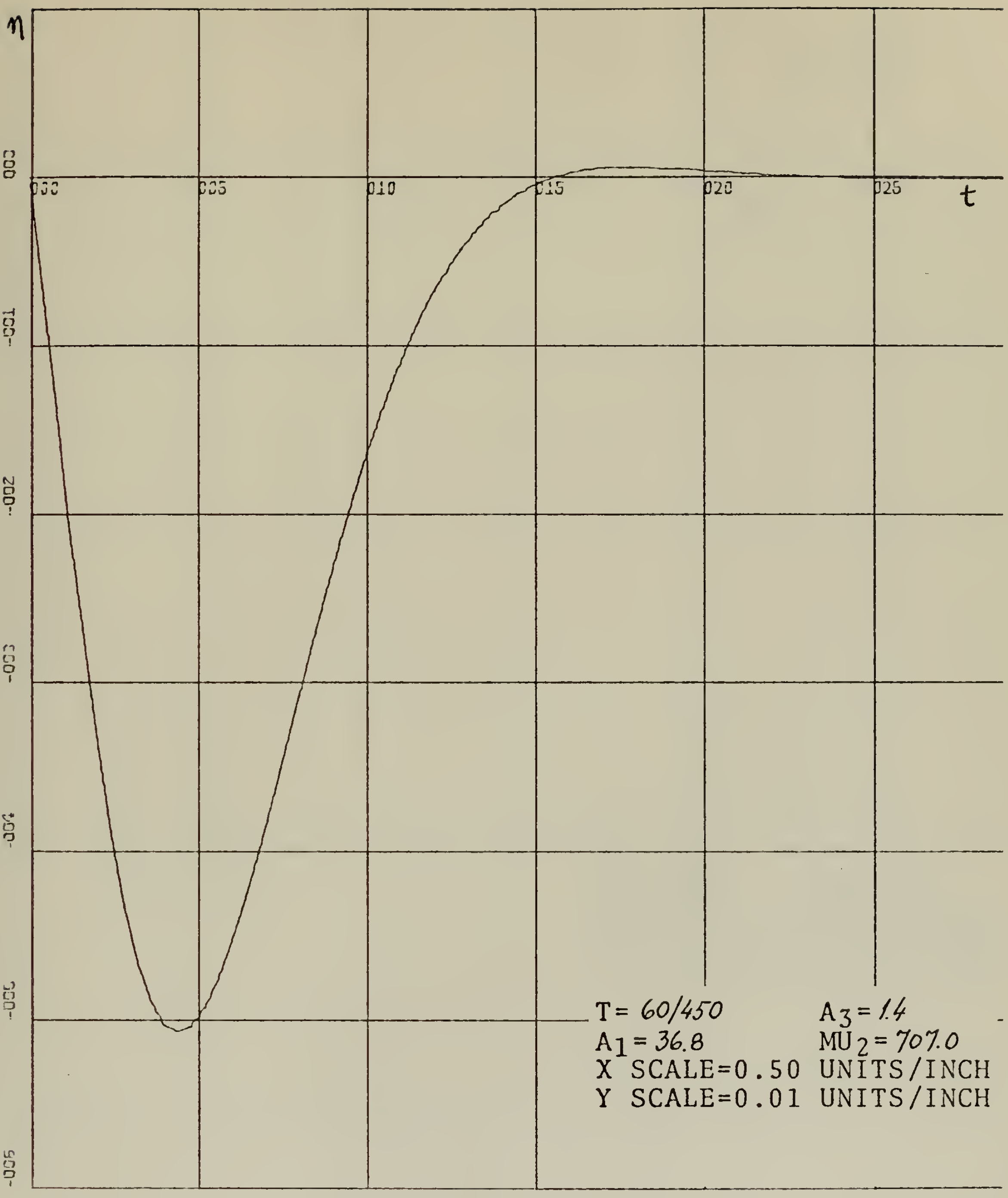


Figure 69. Speed transient of 4-cylinder G.E.

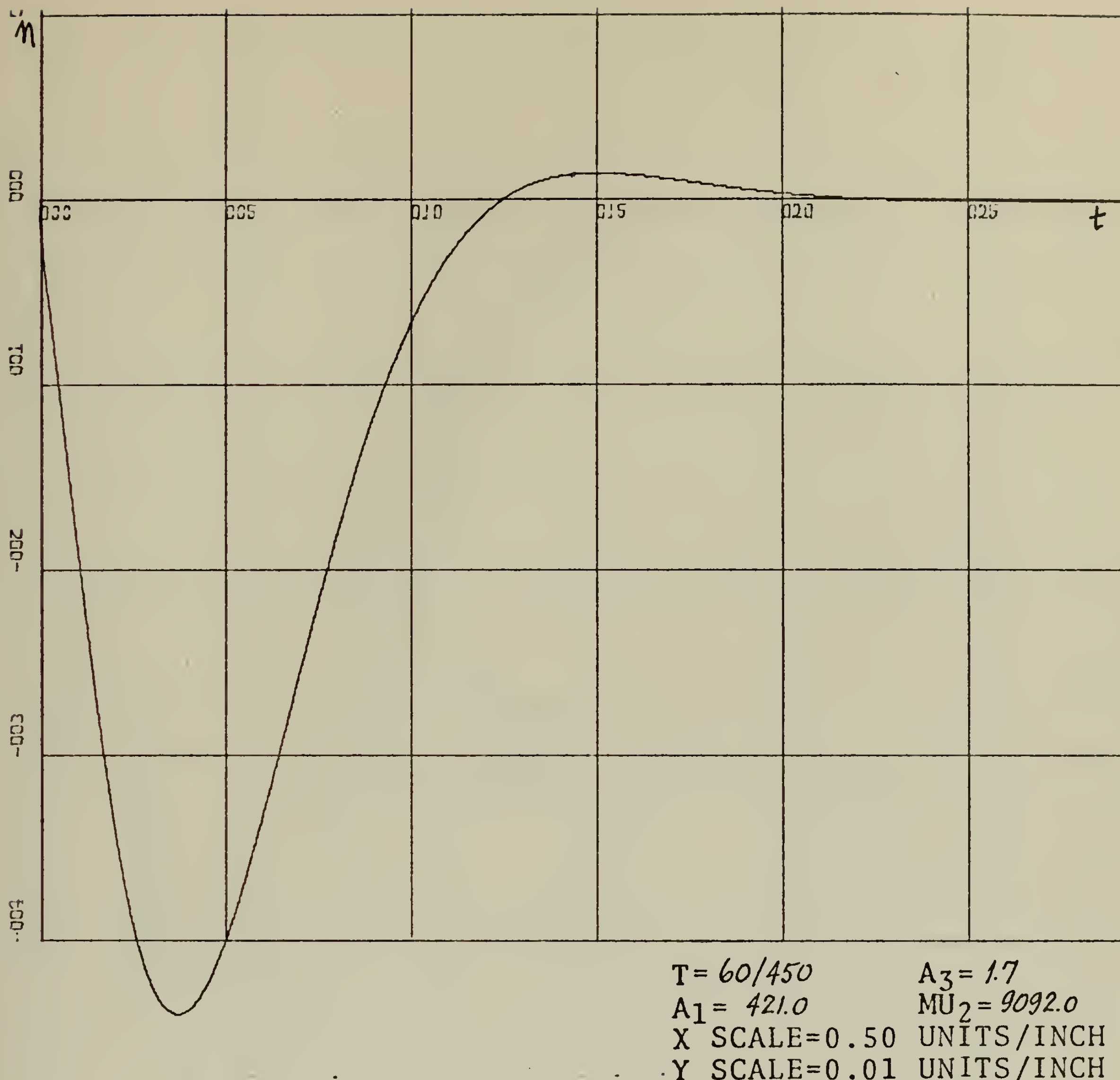


Figure 70. Speed transient of 4-cylinder G.E.

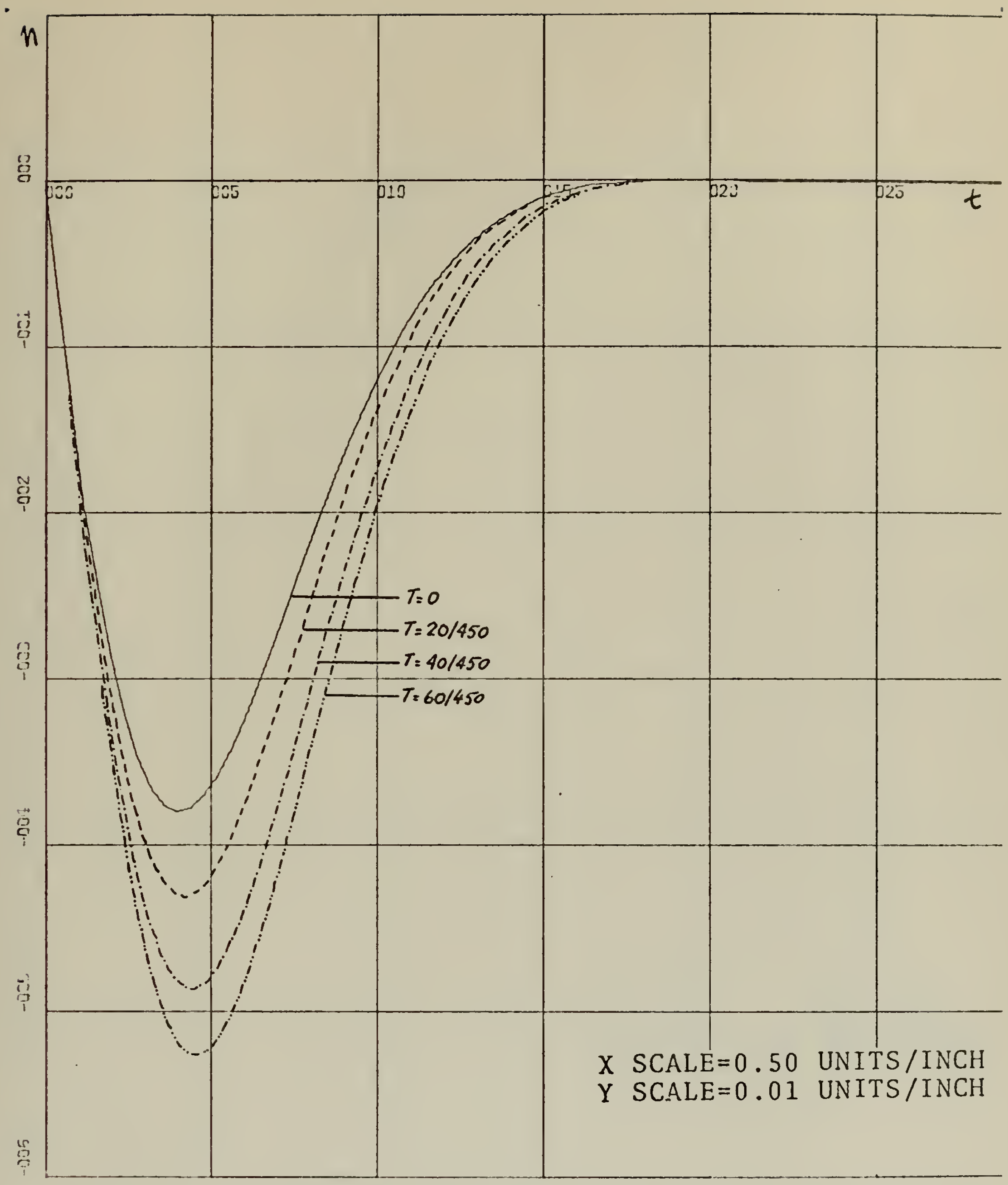


Figure 71. Speed transient of the four systems when the three dominant roots are located at -3 and $-3 \pm j3$.

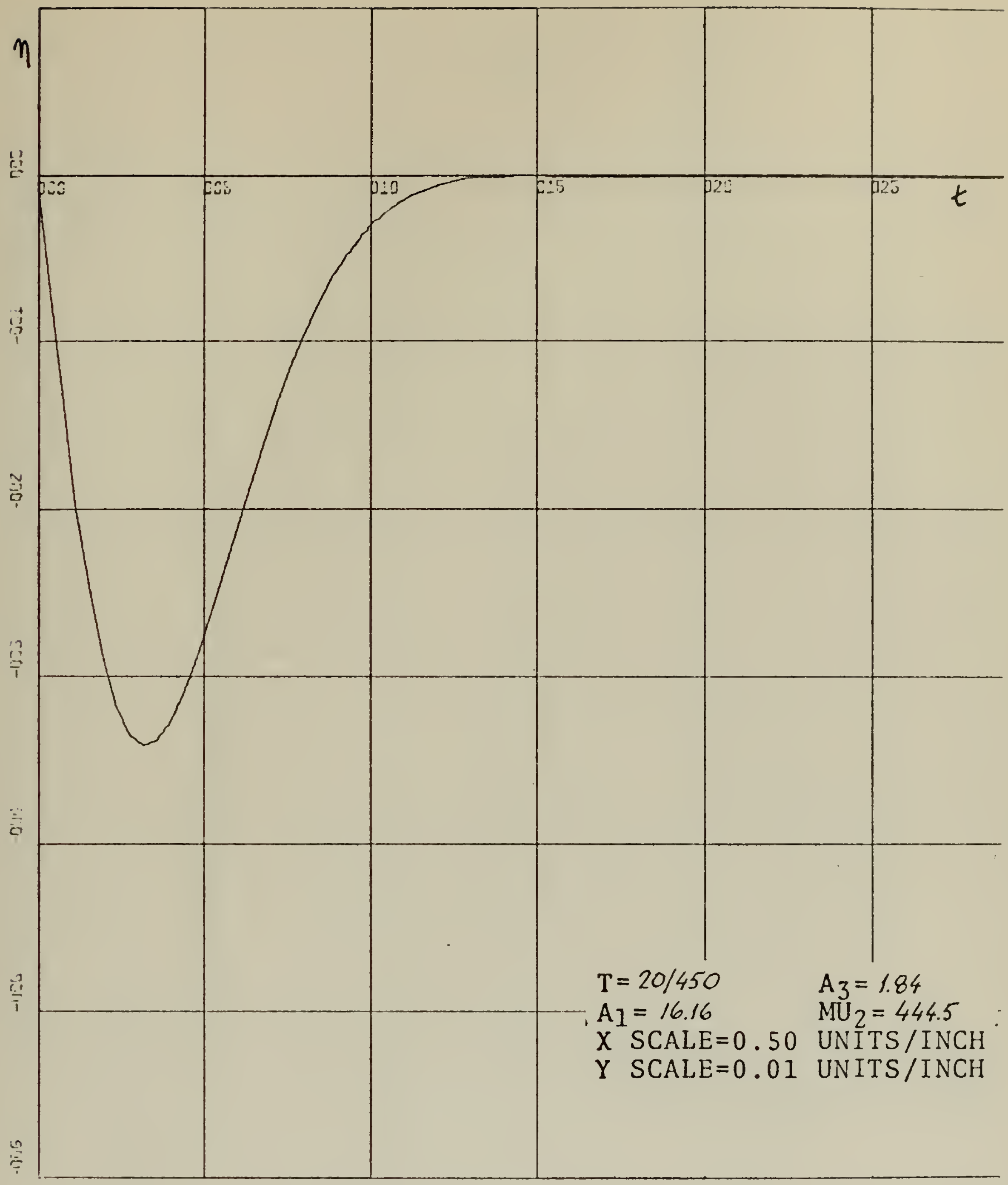


Figure 72. Speed transient of Diesel (Dominant roots of system at -4 and $-4 \pm j4$).

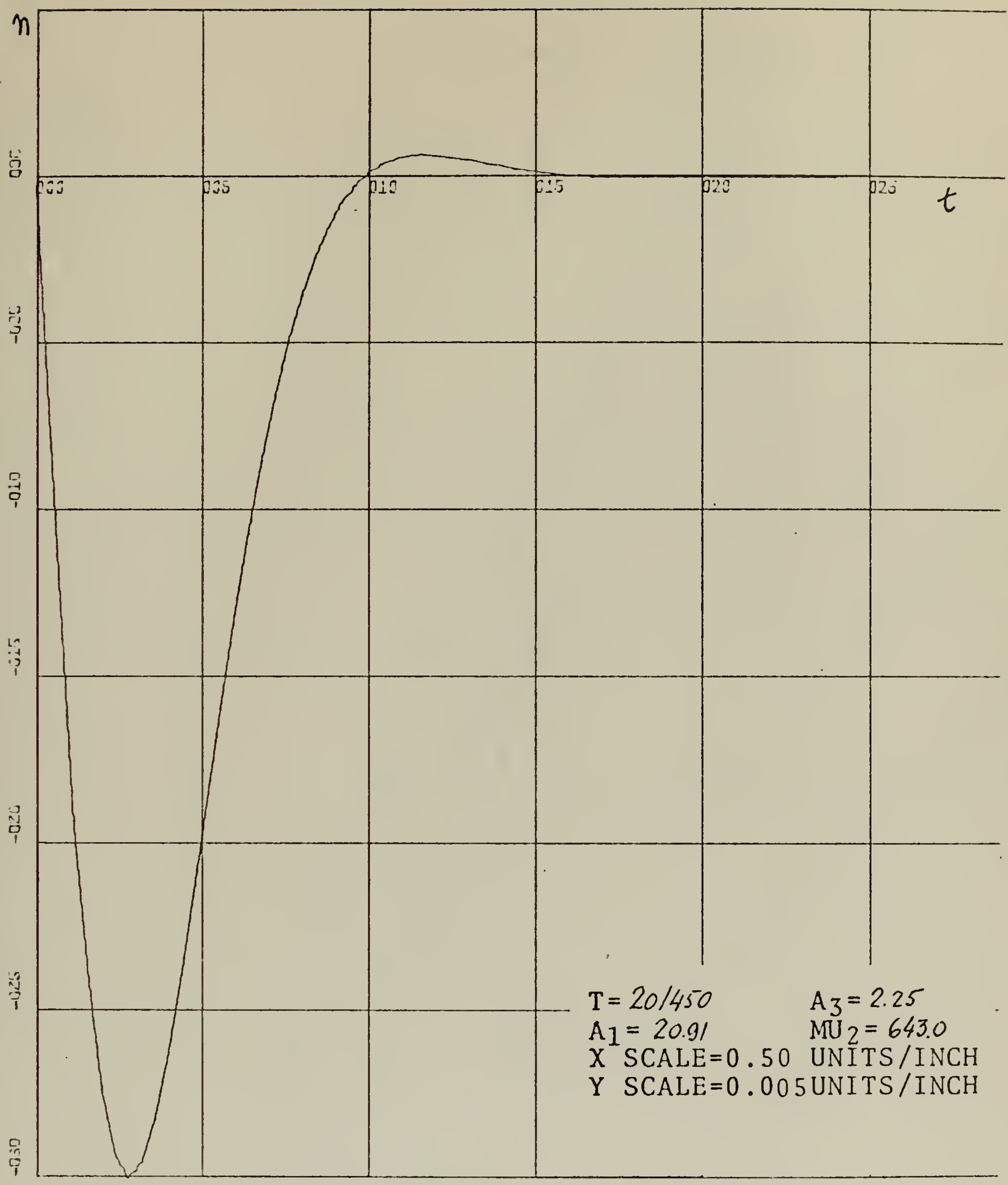


Figure 73. Speed transient of Diesel (Dominant roots of system at −5 and −5 ± j5).

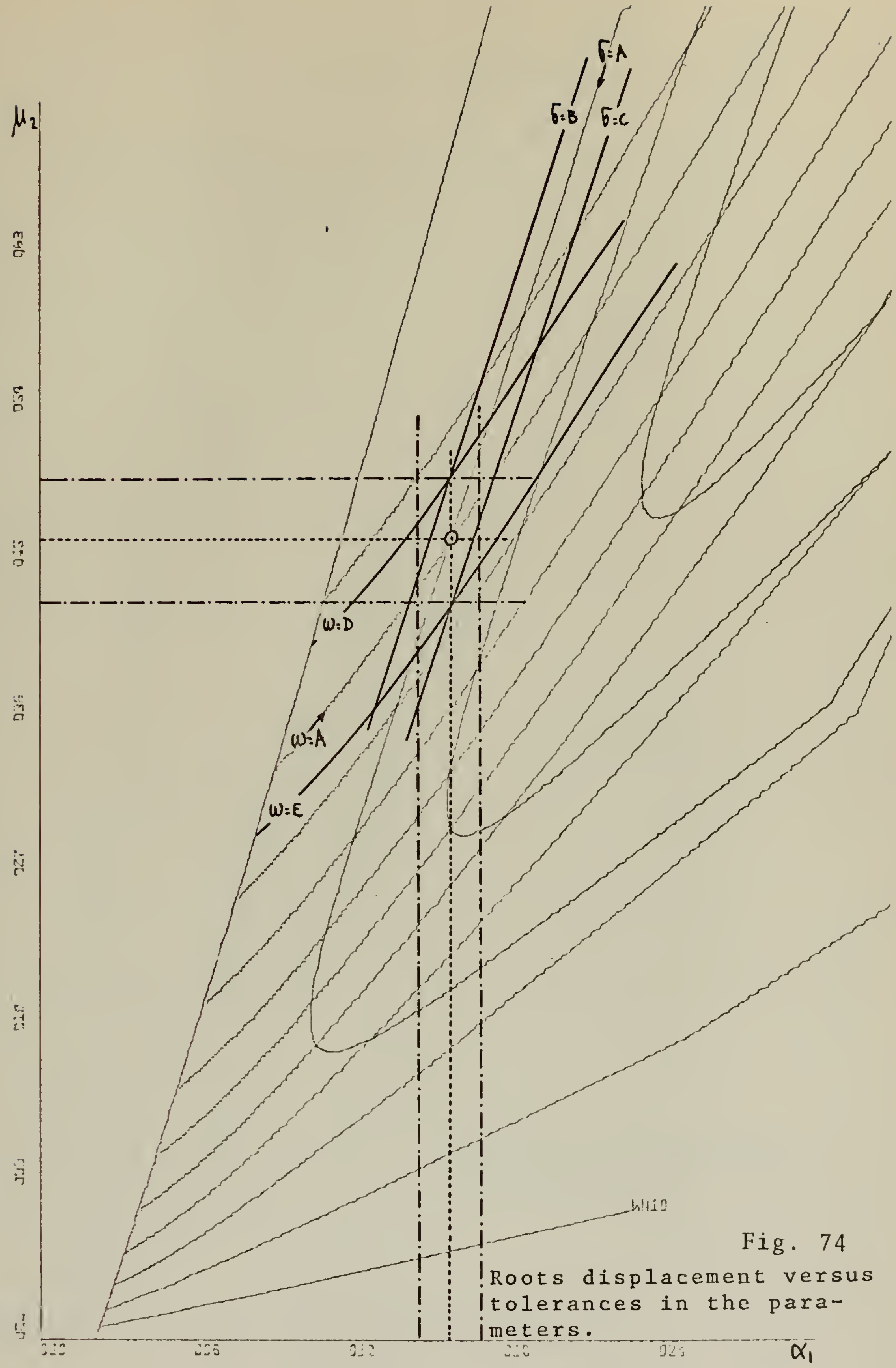
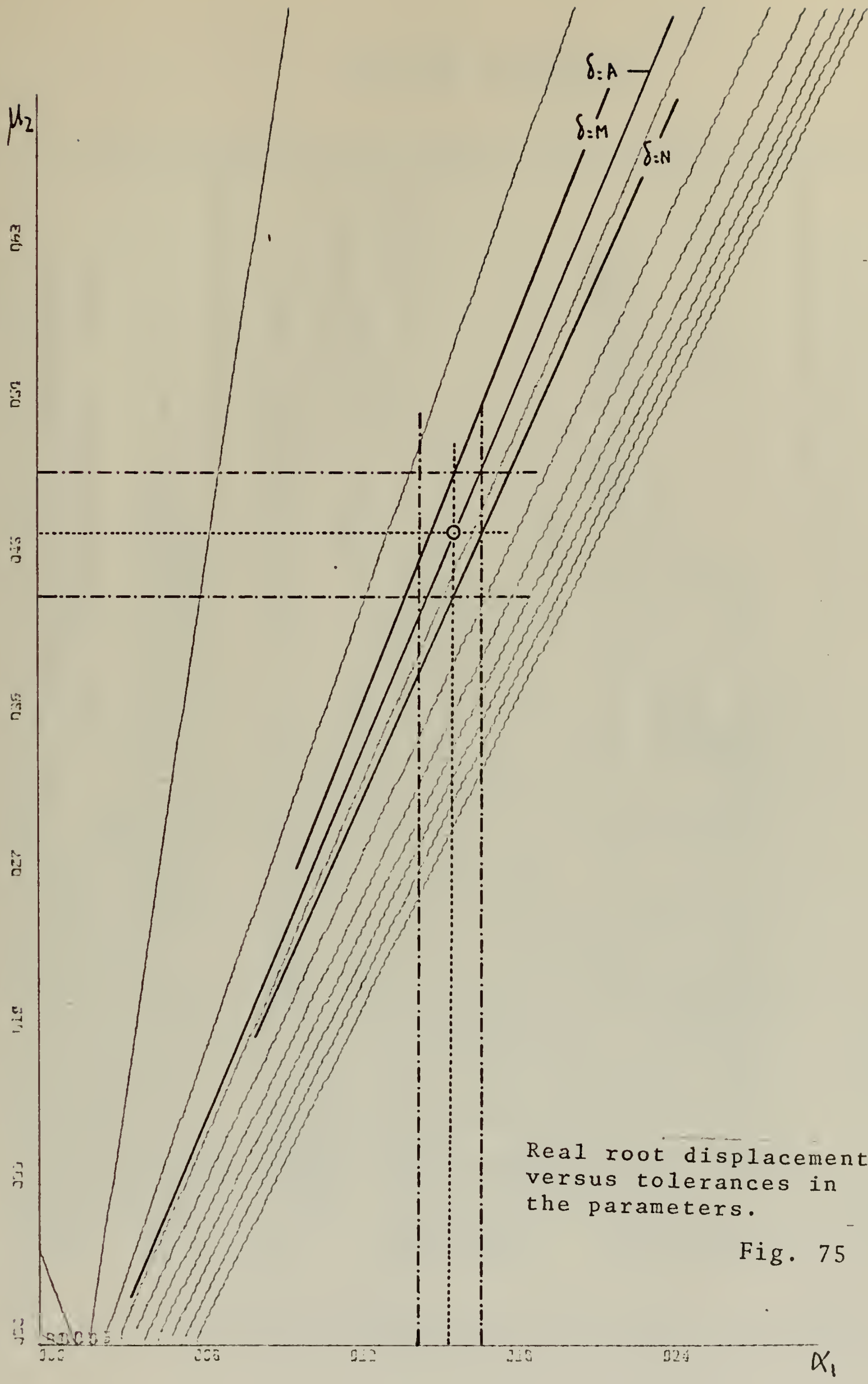


Fig. 74

Roots displacement versus tolerances in the parameters.



Real root displacement
versus tolerances in
the parameters.

Fig. 75

```
//     EXEC  FORTCLGP,REGION.GO=120K
//FORT.SYSIN DD *
C      THIS PROGRAM CALCULATES AND PLOTS CURVES ON THE TWO DIMENSIONAL
C      PARAMETER PLANE OF EITHER:
C         1) CONSTANT SIGMA(FNC OMEGA) AND/OR CONSTANT OMEGA(FNC SIGMA);     *
C         2) DELTA (REAL ROOT) LINES;                                        *
C         3) A COMBINATION OF 1 AND 2 ABOVE;                                 *
C      FOR CHARACTERISTIC EQUATIONS OF THE GOVERNOR DIESEL                   *
C         S**3 + A1*S**2 +  MU2*EXP(-T*S)*S/TM +MU2*A3*EXP(-T*S)/TM          *
C      WHERE THE A AND MU2 COEFFICIENTS ARE CONSTANTS                        *
C                                                                            *
C************* DEFINITION OF SYMBOLS USED FOR INPUT DATA *****************
C                                                                            *
C      NT= TYPE OF OUTPUT CURVES DESIRED                                     *
C         1=CONSTANT SIGMA(FNC OMEGA) AND/OR CONSTANT OMEGA(FNC SIGMA)       *
C         2=DELTA(REAL ROOT) LINES                                           *
C         3=COMBINATION OF 1 AND 2 ABOVE                                     *
C            NOTE: ALL CURVES ARE PLOTTED ON PLAIN PAPER.  IF A 1 INCH GRID  *
C                  IS DESIRED REPLACE ABOVE NT VALUES WITH 11,12,13.         *
C      NN= ROTATIONAL SPEED IN R.P.M.                                        *
C      KR= CONSTANT DEPENDING ON THE ENGINE (DIESEL,2 OR 4 CYCLES GAS) TYPE  *
C      ND=NO. OF DECADES SPANNED BY OMEGA IF NT=1,3.
C      NL= LARGEST SIGMA FOR CONSTANT OMEGA CURVES IF NT=1,3.                *
C      NZ= NO. OF CONSTANT: SIGMA CURVES IF NT=1,3.                          *
C      NW= NO. OF CONSTANT: OMEGA CURVES IF NT=1,3.                          *
C      NS= NO. OF CONSTANT: DELTA CURVES IF NT=2,3.                          *
C      IP= PRINTED OUTPUT DESIRED                                            *
C         1=OUTPUT EVERY ITERATION (300 LINES PER CURVE)                     *
C         2=OUTPUT EVERY TENTH ITERATION (30 LINES PER CURVE)                *
C         3=NO PRINTED OUTPUT                                                *
C      IX= DISTANCE IN INCHES OF THE X-AXIS FROM THE BOTTOM OF THE GRAPH     *
C      IY= DISTANCE IN INCHES OF THE Y-AXIS FROM LEFT SIDE OF THE GRAPH      *
C      TM= MECHANICAL STARTING TIME                                          *
C      WN= START VALUE OF OMEGA FOR CONSTANT SIGMA CURVES                    *
C      XS= X-SCALE IN UNITS PER INCH (CANNOT BE ZERO)                        *
C      YS= Y-SCALE IN UNITS PER INCH (CANNOT BE ZERO)                        *
C      NA3= NO. OF CONSTANT: ALPHA3 PLOTS                                    *
C      LZ= LABELS FOR CONSTANT: SIGMA(NT=1,3) CURVES                         *
C      LW= LABELS FOR CONSTAN: OMEGA(NT=1,3) CURVES                          *
C      LD= LABELS FOR DELTA (REAL ROOT) LINES
C                                                                            *
C************************ DATA CARDS REQUIRED ****************************
C                                                                            *
C      CARD 1  NT,NN,KR,ND,NL,NZ,NW,NS,IP,IX,IY,TM,WN,XS,YS (11I4,4E9.4)     *
C      CARD 2  NA3 (I2)                                                      *
C      CARD 3  CORRESPONDING VALUES OF ALPHA3 FOR EACH NA3 (8E10.5 FORMAT)   *
C      CARD 4  LZ (20A4 FORMAT), USE BLANK CARD IF NT=2                      *
C      CARD 5  LW (20A4 FORMAT), USE BLANK CARD IF NT=2                      *
```

```
C     CARD 6  LD (20A4 FORMAT), USE BLANK CARD IF NT=1,3                    *
C     CARD 7  CORRESPONDING VALUES OF CARD 4 (8E10.5 FORMAT)                *
C     CARD 8  CORRESPONDING VALUES OF CARD 5 (8E10.5 FORMAT)                *
C     CARD 9  CORRESPONDING VALUES OF CARD 6 (8E10.5 FORMAT)                *
C     CARD 10 FIRST LINE OF GRAPH TITLE (COLUMNS 1-48)                      *
C     CARD 11 SECOND LINE OF GRAPH TITLE (COLUMNS 1-48)                     *
C     CARD 13 FIRST LINE OF GRAPH TITLE (COLUMNS 1-48)                      *
C     CARD 14 SECOND .................................                      *
C     ................................................                      *
C         NOTE: FOR EACH PLOT WITH A VALUE OF PARAMETER ALPHA3 IT IS        *
C         REQUIRED TWO CARDS FOR THE NEW GRAPH TITLE (STARTING IN CARD 10)  *
C                                                                           *
C                                                                           *
C****************************************************************************
C                                                                           *
C     LIMITATIONS:                                                          *
C        1. NUMBER OF CURVES LIMITED TO 20 PER TYPE (MAXIMUM OF SIXTY).     *
C           CAN BE INCRESED TO M BY CHANGING DIMENSIONS OF LZ,LW,LD,SZ,WWN, *
C           DELT TO M.                                                      *
C           NUMBER PLOTS FOR DIFFERENTS ALPHA3 LIMITED TO 20. THE   SAME    *
C           RULE CHANGING DIMENSIONS OF ALA TO M.                           *
C                                                                           *
C****************************************************************************
      REAL*8 ITITLE(12)
      REAL LABEL/4H    /,GG(10)/1.0076,1.016,1.0245,1.0312,1.0394,
     *1.0483,1.0568,1.0633,1.071,1.078/,LZ(20),LW(20),
     *LS(20),SZ(20),WWN(20),SIG(20),LD(20),AA(10),ALFA(300),BETA(300),
     <ALA(20),DELT(20),DK(2)
      CALL ERRSET (207,256,-1,1,0,209)                                  TEMP
      WRITE(6,500)
      READ(5,402) NT,NN,KR,ND,NL,NZ,NW,NS,IP,IX,IY,TM,WN,XS,YS
      WRITE(6,501)
      WRITE(6,411) NT,NN,KR,ND,NL,NZ,NW,NS,IP,IX,IY,TM,WN,XS,YS
      READ(5,420) NA3
      WRITE(6,423) NA3
      READ(5,421) (ALA(I),I=1,NA3)
      WRITE(6,422)
      WRITE(6,424) (ALA(I),I=1,NA3)
      NO=2
      IG=0
      IF(NT.LT.10) GO TO 200
      IG=1
      NT=NT-10
  200 READ(5,403)(LZ(I),I=1,NZ)
      WRITE(6,502)
      WRITE(6,404)(LZ(I),I=1,NZ)
      READ(5,403)(LW(I),I=1,NW)
      WRITE(6,503)
```

```
      WRITE(6,404)(LW(I),I=1,NW)
      READ(5,403)(LD(I),I=1,NS)
      WRITE(6,504)
      WRITE(6,404)(LD(I),I=1,NS)
      READ(5,405)(SZ(I),I=1,NZ)
      WRITE(6,505)
      WRITE(6,406)(SZ(I),I=1,NZ)
      READ(5,405)(WWN(I),I=1,NW)
      WRITE(6,506)
      WRITE(6,406)(WWN(I),I=1,NW)
      READ(5,405)(DELT(II),II=1,NS)
      WRITE(6,507)
      WRITE(6,406)(DELT(II),II=1,NS)
      DO 300 I=1,NA3
      ALFA3=ALA(I)
      WRITE(6,526)
      WRITE(6,425) ALFA3
      READ(5,400)ITITLE
      WRITE(6,413)ITITLE
      MOD=1
      RPM=NN
      RFAC=KR
      T=RFAC/RPM
      XLIMP=(9.2-IY)*XS
      YLIMP=(15.2-IX)*YS
      XLIMN=-IY*XS
      YLIMN=-IX*YS
      GO TO (103,301,103),NT
  103 DO 146 JJ=1,2
      GO TO (204,205),JJ
  204 IF(NZ.EQ.0) GO TO 146
      NI=NZ
      G=GG(ND)
      GO TO 206
  205 IF(NW.EQ.0) GO TO 146
      NI=NW
      DEL=NL/300.0
      WRITE(6,526)
  206 DO 145 K=1,NI
      GO TO (104,207),JJ
  104 W=WN
      SIGZ=SZ(K)
      GO TO 105
  207 SIGZ=0.0
      W=WWN(K)
  105 LL=0
      LLL=0
      LLLL=0
```

```
      DO 242 L=1,300
      IPP=0
  106 IF(L.GT.1) GO TO 208
      GO TO (306,307),JJ
  306 WRITE(6,511)
      WRITE(6,513)
      GO TO 208
  307 WRITE(6,512)
      WRITE(6,513)
  208 CALL RODRI(SIGZ,W,T,TM,ALFA3,DK,KS)
      KS=KS+1
      GO TO (117,116),KS
  116 WRITE(6,407)SIGZ,W
      GO TO 240
  117 ATEMP=DK(1)
      BTEMP=DK(2)
      IF(L.GT.1) GO TO 217
      AMAX=ATEMP
      AMIN=ATEMP
      BMAX=BTEMP
      BMIN=BTEMP
  217 IF(ATEMP.GE.AMIN) GO TO 317
      AMIN=ATEMP
  317 IF(ATEMP.LE.AMAX) GO TO 417
      AMAX=ATEMP
  417 IF(BTEMP.GE.BMIN) GO TO 617
      BMIN=BTEMP
  617 IF(BTEMP.LE.BMAX) GO TO 218
      BMAX=BTEMP
  218 IF(XLIMN-ATEMP)119,119,123
  119 IF(ATEMP-XLIMP)120,120,123
  120 IF(YLIMN-BTEMP)121,121,123
  121 IF(BTEMP-YLIMP) 122,122,123
  122 LL=LL+1
      LLL=LLL+1
      GO TO (138,234,140),IP
  123 GO TO (124,124,240),IP
  124 IPP=1
      GO TO (138,234,142),IP
  127 GO TO (138,138,142),IP
  234 IF(L.EQ.1.OR.LLLL.EQ.10) GO TO 235
      GO TO 139
  235 LLLL=0
  138 WRITE(6,409)ATEMP,BTEMP,SIGZ,W
      GO TO (139,139,240),IP
  139 IF(IPP.EQ.1)GO TO 240
  140 ALFA(LL)=ATEMP
      BETA(LL)=BTEMP
```

```
240 IF(JJ.EQ.1)GO TO 141
    SIGZ=SIGZ+DEL
    GO TO 142
141 W=W*G
142 LLLL=LLLL+1
    GO TO 242
242 CONTINUE
    IF(LL.LT.2)GO TO 144
    IF(JJ.EQ.1)GO TO 143
    CALL DRAW(LL,ALFA,BETA,MOD,0,LW( K),ITITLE,XS,YS,IX,IY,2,2,
   *5,8,IG,LAST)
    WRITE(6,519) W,LL
    WRITE(6,525) AMAX,BMAX,AMIN,BMIN
    MOD=2
    GO TO 145
143 CALL DRAW(LL,ALFA,BETA,MOD,0,LZ(K),ITITLE,XS,YS,IX,IY,2,2,
   *5,8,IG,LAST)
    WRITE(6,520) SIGZ,LL
    WRITE(6,525) AMAX,BMAX,AMIN,BMIN
    MOD=2
    GO TO 145
144 GO TO (244,344),JJ
244 WRITE(6,514) SIGZ
    WRITE(6,525) AMAX,BMAX,AMIN,BMIN
    GO TO 145
344 WRITE(6,521) W
    WRITE(6,525) AMAX,BMAX,AMIN,BMIN
145 CONTINUE
146 CONTINUE
165 IF(MOD.EQ.1) GO TO 166
    ALFA(1)=0.0
    ALFA(2)=1.0
    BETA(1)=0.0
    BETA(2)=0.0
    CALL DRAW(2,ALFA,BETA,3,0,LABEL,ITITLE,XS,YS,IX,IY,2,2,
   *5,8,0,LAST)
302 IF(NT.EQ.1) GO TO 300
301 WRITE(6,526)
    XDELTA=XLIMP-XLIMN
    YDELTA=YLIMP-YLIMN
    XD=XDELTA/300.
    YD=YDELTA/300.
    MODE=1
    MM=0
    IF((NS-1).EQ.0) MODE=0
    DO 3 II=1,NS
    WRITE(6,510)
    WRITE(6,515)
```

```
      DETEM=DELT(II)
      ALFA1=XLIMN
      XT=-T*DETEM
      XE=EXP(XT)
      XK=TM*XE*DETEM**2
      MMM=1
      IF(ALFA3.NE.DETEM) GO TO 8
      DO 6 L=1,300
      MMM=MMM+1
      DD=YD*L
      BETEM=YLIMN+DD
      BETA(L)=BETEM
      ALFA(L)=DETEM
      GO TO (20,21,6),IP
   20 MMM=1
      WRITE(6,408) DETEM,BETEM,DETEM
      GO TO 6
   21 IF(MMM.EQ.10) GO TO 20
      MMM=MMM+1
    6 CONTINUE
      CALL DRAW(300,ALFA,BETA,MODE,0,LD(II),ITITLE,XS,YS,IX,IY,2,2,5,
     *8,IG,LAST)
      GO TO 5
    8 XKK=DETEM-ALFA3
      SLOPE=XK/XKK
      MP=0
      DO 2 J=1,300
      BETA1=SLOPE*(ALFA1-DETEM)
      IF(YLIMN-BETA1) 10,10,31
   10 IF(BETA1-YLIMP) 15,15,31
   15 MP=MP+1
      ALFA(MP)=ALFA1
      BETA(MP)=BETA1
      GO TO (30,31,4),IP
   30 MMM=1
      WRITE(6,408) ALFA1,BETA1,DETEM
      GO TO 4
   31 IF(MMM.EQ.10) GO TO 30
      MMM=MMM+1
    4 ALFA1=ALFA1+XD
      JJ=J+1
    2 CONTINUE
      WRITE(6,516) DETEM,MP
      CALL DRAW(MP,ALFA,BETA,MODE,0,LD(II),ITITLE,XS,YS,IX,IY,2,2,5,
     *8,IG,LAST)
    5 MM=MM+1
      IF(MODE.EQ.3) GO TO 7
      IF((NS-MM).GT.1) GO TO 11
```

```
      MODE=3
      GO TO 3
   11 IF(MP.GE.2) MODE=2
    3 CONTINUE
      IF(MM.NE.0) GO TO 7
      WRITE(6,518)
      GO TO 300
    7 WRITE(6,522) LAST
      GO TO 300
  166 WRITE(6,518)
      GO TO 302
  300 CONTINUE
  400 FORMAT(6A8)
  401 FORMAT(2X,6A8)
  402 FORMAT(11I4,4E9.4)
  403 FORMAT(20A4)
  404 FORMAT(2X,20(A4,2X))
  405 FORMAT(8E10.5)
  406 FORMAT(2X,8E14.5)
  407 FORMAT(T7,'POSSIBLE SINGULAR POINT',2E14.5,T64,'NO ROOTS;'
     *T73,'POINT NOT PLOTTED ON GRAPH')
  408 FORMAT(3E14.5)
  409 FORMAT(4E14.5)
  410 FORMAT(58X,6E12.4)
  411 FORMAT(11I4,1X,4E14.4)
  412 FORMAT(76X,4E12.4)
  413 FORMAT(///,10X,6A8)
  420 FORMAT(I2)
  421 FORMAT(8E10.5)
  422 FORMAT(///,T3,'VALUES OF PARAMETER ALPHA3:',/)
  423 FORMAT(///,T3,'THE NUMBER OF GRAPHS=',I2)
  424 FORMAT(2X,8E14.5)
  425 FORMAT(///,T30,'PARAMETER ALPHA3=',E14.5,/)
  500 FORMAT('1',T6,'THE INPUT DATA IS:',////)
  501 FORMAT(//,3X,2HNT,1X,2HNN,2X,2HKR,2X,2HND,2X,2HNL,2X,2HNZ,2X,
     <2HNW,2X,2HNS,2X,2HIP,2X,2HIX,2X,2HIY,6X,2HTM,T65,'START OMEGA',
     <T81,'X-SCALE',T95,'Y-SCALE',/)
  502 FORMAT(///,T3,'LABELS FOR CONSTANT SIGMA',/)
  503 FORMAT(///,T3,'LABELS FOR CONSTANT OMEGA',/)
  504 FORMAT(///,T3,'LABELS FOR CONSTANT DELTA',/)
  505 FORMAT(///,T3,'VALUES FOR CONSTANT SIGMA CURVES',/)
  506 FORMAT(///,T3,'VALUES FOR CONSTANT OMEGA CURVES',/)
  507 FORMAT(///,T3,'VALUES FOR CONSTANT DELTA CURVES',/)
  510 FORMAT(///,T14,'CONSTANT DELTA CURVE',/)
  511 FORMAT(///,T19,'CONSTANT SIGMA CURVE',/)
  512 FORMAT(///,T19,'CONSTANT OMEGA CURVE',/)
  513 FORMAT(T7,'ALPHA1',T22,'MU2',T34,'SIGMA',T49,'OMEGA',//)
  514 FORMAT(T8,'CURVE FOR CONSTANT  SIGMA = ' ,E13.5,T53,'WAS NOT PLO
```

```
     *TTED',/,T8,'AS LESS THAN TWO POINTS WERE GENERATED WITHIN SPECIFIE
     *DGRAPH RANGE',/)
 515 FORMAT(T7,'ALPHA1',T22,'MU2',T34,'DELTA',//)
 516 FORMAT(T8,'FOR THE CURVE OF CONSTANT DELTA=',E13.5,/,I10,T12,
     <'POINTS OUT OF A POSSIBLE 300 LIE WITHIN SPECIFIED GRAPH RANGE',/)
 518 FORMAT(////,T3,'THERE WILL BE NO OUTPUT GRAPH PLOTTED AS NONE',/,
     *T3,'OF THE POINTS GENERATED BY THE PROGRAM LIE',/,
     *T3,'WITHIN THE SPECIFIED GRAPH REGION. CHECK THE VALUES OF',/,
     *T3,'XS AND YS AS WELL AS IX AND IY ON DATA CARD 3',/)
 519 FORMAT(T8,'FOR THE CURVE OF CONSTANT OMEGA     =      ',E13.5,
     */,I10,T12,
     *'POINTS OUT OF A POSSIBLE 300 LIE WITHIN SPECIFIED GRAPH RANGE',/)
 520 FORMAT(T8,'FOR THE CURVE OF CONSTANT SIGMA     =    ',E13.5,
     */,I10,T12,
     *'POINTS OUT OF A POSSIBLE 300 LIE WITHIN SPECIFIED GRAPH RANGE',/)
 521 FORMAT(T8,'CURVE FOR CONSTANT OMEGA     =        ',E13.5,T57,'WAS NOT
     * PLOTTED',/,T8,'AS LESS THAN TWO POINTS WERE GENERATED WITHIN SPEC
     *IFIED GRAPH RANGE',/)
 522 FORMAT(///,5X,'LAST=',I2,//)
 525 FORMAT(T8,'MAXIMUM AND MINIMUM GENERATED ALPHA1 & MU2 VALUES WERE:    0
     *',/,T8,'AMAX=',E10.3,T24,'BMAX=',E10.3,T40,'AMIN=',E10.3,T56,
     *'BMIN=',E10.3,/)
 526 FORMAT('1')
     STOP
     END


     SUBROUTINE RODRI(XSIGZ,XW,XT,XTM,XALFA,DK,KS)
     IMPLICIT REAL*8(X-Y)
     DIMENSION DK(2)
     KS=0
     XZ=XW*XT
     XX=XSIGZ*XT
     XA=XSIGZ**2-XW**2
     XEX=DEXP(XX)/XTM
     XS=DSIN(XZ)
     XC=DCOS(XZ)
     XB=XEX*(XW*XS+XC*(XALFA-XSIGZ))
     XAA=-2*XSIGZ*XW
     XBB=XEX*(XW*XC-XS*(XALFA-XSIGZ))
     XD1=XSIGZ**3-3*XSIGZ*XW**2
     XD2=XW**3-3*XW*XSIGZ**2
 801 XDETER=XA*XBB-XB*XAA
     IF(XDETER.EQ.0.0) GO TO 4
     DK(1)=(XD1*XBB-XD2*XB)/XDETER
     DK(2)=(XA*XD2-XAA*XD1)/XDETER
     GO TO 5
   4 KS=1
```

```
5  RETURN
   END
```

```
//  EXEC CSMP360
//CSMP.SYSIN DD *
*  CSMP PROG. FOR SYSTEM TESTING WITH DIF. SETS OF ALFA1,ALFA3,MU2
*  TESIS
*
*
         INITIAL
         PARAM ALFA1=9.8,ALFA3=1.7,MU2=242.0,TM=5.88,RFAC=0.0,RPM=450.
         INCON CIC=0.0
         DYNAMIC
         T=RFAC/RPM
         S=STEP(0.0)
         B=S-H
         C=-(1.0/TM)*B
         D=INTGRL(CIC,C)
         E=-(MU2*ALFA3/ALFA1)*D
         O=1.0/ALFA3
         P=1.0/ALFA1
         F=LEDLAG(O,P,E)
         G=INTGRL(CIC,F)
         H=DELAY(100,T,G)
         TIMER FINTIM=4.,DELT=4.E-02,PRDEL=4.E-02
         TITLE ENGINE-GOVERNOR SYSTEM WITH TIME DELAY
         PRTPLOT D
         PREPAR TIME,D
         END
         STOP
ENDJOB
```

COMPUTER PROGRAM #2

```
//   EXEC CSMP360
//CSMP.SYSIN DD *
* THIS IS A CSMP PROGRAM FOR ANONLINEAR SYSTEM
*   TESIS
*
*
INITIAL
      PARAM ALFA1=9.2,ALFA3=1.55,MU2=216.9,TM=5.88,RFAC=0.0,RPM=450.
DYNAMIC
      ALFA=RAMP(0.0)
      DELTA=-ALFA
      T=RFAC/RPM
      A=DELTA**3
      B=ALFA1*DELTA**2
      F=-T*DELTA
      E=EXP(F)
      C=(MU2/TM)*E*DELTA
      D=(MU2*ALFA3/TM)*E
      G=A+B+C+D
      TIMER FINTIM=5.0,DELT=5.0E-02,PRDEL=5.0E-02
      TITLE CALCULATION OF DELTA
      PRTPLOT G
      END
      STOP
ENDJOB
```

COMPUTER PROGRAM #3

LIST OF REFERENCES

1. Han, K. W., G. J. Thaler, "Control Systems Analysis and Design with a Parameter Space Method," IEEE Transactions on Automatic Control, V. AC-11, p. 560, 563, July 1966.

2. Eisenberg, L. "Stability of Linear Systmes with Transport Lab," IEEE Transactions on Automatic Control, V. AC-11, p. 247-254, April 1966.

3. Chen, Y. Y. K., K. W. Han, G. J. Thaler, "Analysis of Nonlinear Control Systems with Transport Lab,"IEEE Transactions on Industry and General Applications, V. IGA-7, p. 576-579, September/October 1971.

4. Leum, Mark, George Parker, "Ideal Governor Settings in the Presence of Engine Dead Time," Memo circulated within Woodward Governor Company, October 1958.

INITIAL DISTRIBUTION LIST

		No. Copies
1.	Defense Documentation Center Cameron Station Alexandria, Virginia 22314	2
2.	Library, Code 0212 Naval Postgraduate School Monterey, California 93940	2
3.	Professor G. J. Thaler, Code 52Tr Department of Electrical Engineering Naval Postgraduate School Monterey, California 93940	3
4.	Professor M. L. Wilcox, Code 52Wx Department of Electrical Engineering Naval Postgraduate School Monterey, California 93940	1
5.	Lieutenant Francisco J. F. Neto Direccao de Armas Navais Alfeite, Portugal	2
6.	Professor D. R. Towill Department of Mechanical Engineering and Engineering Production University of Wales Institute of Science and Technology Cardiff, Wales, U.K.	1
7.	Professor D. D. Siljak Electrical Engineering Department University of Santa Clara Santa Clara, California	1
8.	Professor L. Eisenberg Moore School of Electrical Engineering University of Pennsylvania Philadelphia, Pennsylvania	1
9.	Mr. Mark Leum Woodward Governor Company 5001 North Second Street Rockford, Illinois 61101	1

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Naval Postgraduate School Monterey, California 93940	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE

Parameter Plane Studies of Engine-Governor Systems with Time Delay

4. DESCRIPTIVE NOTES (Type of report and, inclusive dates)

Master's Thesis, December 1972

5. AUTHOR(S) (First name, middle initial, last name)

Francisco J. F. Neto

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
December 1972	124	4

8a. CONTRACT OR GRANT NO.	9a. ORIGINATOR'S REPORT NUMBER(S)
b. PROJECT NO.	
c.	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)
d.	

10. DISTRIBUTION STATEMENT

Approved for public release; distribution unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	Naval Postgraduate School Monterey, California 93940

13. ABSTRACT

The generally difficult problem of a system with time delay is studied by using parameter space methods and computer analysis. The solution is proved to be very easily obtained and the results presented are extendable under certain circumstances to other systems and useful in other fields of analysis.

DD FORM 1 NOV 65 1473 (PAGE 1)

S/N 0101-807-6811

Security Classification

A-31408

Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Parameter Plane						
Time Delay						
Transport Lag						
Engine Governors						
Governors						

DD FORM 1 NOV 65 1473 (BACK)

S/N 0101-807-6821

Security Classification A-31

Thesis 143218
N425 Neto
c.1 Parameter plane studies of engine-governor systems with time delay.